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ROYAL AIRCRAFT ESTABLISHMENT

TECHNICAL NOTE No. SPACE 60

THE PERFORMANCE CAPABILITY OF THE BLACK KNIGHT ROCKET IN VARIOUS ROLES

by

A. P. Waterfall, B.Sc., and L. W. Parkin, M.Sc.

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SUMMARY

Performance studies made over a number of years for Black Knight are collected together in one document. Most of these calculations are concerned with the vehicle used as a re-entry test vehicle or as a sounding rocket. The note summarises the results and serves to illustrate the improvements in performance which have been made; results are presented in a way which enables the effects on performance of changes in the basic vehicle characteristics to be readily predicted.

Also included are the results of specific studies made to determine the feasibility of using Black Knight as a ballistic missile, as the first stage of a satellite launcher or as a launcher for a hypersonic glider.

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1 INTRODUCTION

Black Knight has been in use as a re-entry test vehicle and sounding rocket for a number of years, during which time there has been a constant need for performance estimates. As a result a number of digital computer programmes have been developed which calculate the position and velocity co-ordinates as a function of time and present them in a form most useful for trials purposes.

The programmes have been used to estimate the performance of a variety of configurations to meet specific trials requirements. In addition, numerous calculations of a more general nature have been made to investigate the effects of changes in fuel weights, thrust etc. The majority of these calculations have been concerned with the use of Black Knight in roles for which all trajectories are near vertical; they have covered single, two-stage and three-stage vehicles. The third stage propulsion consists of a cluster of small rockets to separate the re-entry head from the second stage and adds nothing to the performance.

A limited number of calculations have been made for the use of Black Knight for other purposes, namely launching a glider at hypersonic speeds, launching a satellite and as an intermediate range ballistic missile. For these roles the powered flight path has to be turned from vertical at launch, to some value between 45° elevation and horizontal at all-burnt.

Most of these studies have had only limited circulations; this Note summarises the results and also serves to illustrate the improvements in performance which have been made and the expectations for the future.

2 SUMMARY OF CONFIGURATIONS AND CASES CONSIDERED

The following is a summary of the configurations considered; only brief descriptions are given - further details can be found by reference to more descriptive reports[1,2,3,4,5].

2.1 Single stage vehicle with near vertical trajectory

Whether considering single or multistage vehicles, estimates of the performance of the first stage are a prime necessity. The important parameters defining performance are the velocity V_o and height H_o at burn-out, as a function of the total all-burnt weight. The all-burnt weight includes the weight of experimental equipment or payload. Performance is of course affected by the thrust and specific impulse of the engine, the weight of propellant at launch and the drag of the body; all these factors are considered. "Near vertical trajectory" refers to the fact that in a normal Black Knight launching the significant elevation angle lies between $86\frac{1}{2}$ and $88\frac{1}{2}$ degrees.

In flights associated with "re-entry" problems it is usually more convenient to specify overall performance in terms of the velocity at some fixed height during the descent. The reference height used is 200,000 feet and the velocity V_{E1} at this height on the descent is referred to as the re-entry velocity. By assuming a vacuum trajectory, V_{E1} is readily derived from V_o and H_o, the velocity and height at burn-out on the ascent.

2.1.1 36" vehicle

All firings[6] of Black Knight up-to-date have been made with a 36" diameter vehicle and the majority of these used the original mark of engine, the Gamma 201 with sea level thrust of 16,500 to 17,000 lb weight. One of the disadvantages of this engine has proved to be the lack of fuel and propellant mixture ratio control, making it impossible to accurately predict the amounts of kerosene and H.T.P. to be loaded.

An improved version of this engine, the Gamma 301, has such control. In its present form it is capable of a sea level thrust of 21,600 lb and although at present this is the maximum thrust contemplated for a 36" vehicle, calculations have covered a range of thrusts up to 25,000 lb weight, with take-off weights of up to 20,000 lb.

2.1.2 54" vehicle

Higher thrusts up to 25,000 lb weight can be obtained from the Gamma 301 engine with only minor modifications. The combustion chambers remain sensibly the same, but a new pump is required to provide the additional propellant flow rate. In order to take advantage of this thrust it is necessary to increase the take-off weight by utilising more propellant. The tank capacity can be increased by lengthening the vehicle, but this is undesirable giving a particularly long vehicle when using a second stage. It is more convenient to increase the tank diameter. A convenient configuration has a main body of 54" diameter. This gives a higher drag than the longer vehicle; a limited number of cases have been calculated to show the order of this effect - these indicate that to a very close approximation the re-entry velocity is reduced by 500 ft/sec or less.

2.2 Two stage vehicle (second stage fired downwards)

All two stage Black Knights to date have fired the second stage on the downward leg of a near vertical first stage trajectory. The reason for preferring the downward leg to the upward leg is to reduce dispersion of the second stage. It will be shown in Section 3 that to achieve high performance as well as low dispersion the second stage should be fired as late and as low as possible; the only restriction is that all-burnt must be reached above an altitude of 200,000 ft when the re-entry experiment begins. Performance is measured by the velocity V_{E2} at 200,000 ft on the descent. In this case generalisation and presentation of results is made more difficult because of the additional variables involved.

The second stage motor used so far has been the Cuckoo I, having a total weight of 530 lb, a vacuum specific impulse I_v of 220 secs and a mass fraction (propellant weight/total weight) of only 0·755, which is poor by present day standards. This was the only suitable British motor available when the two-stage Black Knight firings started and so it had to be used in spite of its comparatively poor mass fraction. An improved form of this motor, the Cuckoo II, has been developed by R.P.E. Westcott for Black Knight for use in the Dazzle programme; the vacuum specific impulse has been increased to 250 secs and the mass fraction to 0·85, with no appreciable change in propellant weight.

A new and larger motor, the 24" Kestrel, has been designed especially for Black Knight use, and is at present under development. This will have a vacuum specific impulse of 270 secs and a mass fraction of 0·93 for a total weight of 1160 lb and naturally gives a great improvement in performance. Calculations have been made with all three motors in combination with the two different first-stages, although it is now unlikely that the Cuckoo II and Kestrel will be used in the simple two-stage role.

2.3 Three stage vehicle

The need for the third stage arose from a decision to measure the radiation from the head and its wake during re-entry into the earth's atmosphere. To ensure that any radiation and radar echoes recorded on the ground based instrumentation originates from the head and not the second stage, a separation of at least 20,000 feet at 300,000 feet is required.

The first method considered for obtaining this separation was the fitting of retro-rockets to the second stage. This was rejected because the efflux would contaminate the head, and it was decided to add a small third stage to the two stage vehicle described above. This third stage, firing immediately after the second stage burn-out, propels away from the second stage a sabot in which sits the re-entry head. The travel of the sabot is restricted by a nylon lanyard tying it to the second stage. When the sabot motion is arrested the head continues to move away from the second stage and sabot with the relative velocity imparted to it by the third stage rockets. The possibility of retarding the sabot by its own retro-rockets when the velocity is adequate has also been considered. At present, engineering considerations limit the lanyard method to a maximum separation velocity of 400 ft/sec; there is no real limit to the other method using retro-rockets on the sabot but 700 ft/sec can be taken as a practical value.

In the first three stage vehicle to be fired the third stage was powered by a cluster of 26 IMP VI thrust units which, with a total impulse of 1700 lb secs gives 400 ft/sec separation velocity with a 110 lb head. These 26 IMP units will be replaced in future firings by 4 IMP X thrust units, again specially developed for Black Knight use, which give the same total impulse for less weight and so improved overall performance. For 700 ft/sec separation velocity 10 IMP X units could be used, 2 of them being used to retard the sabot. The performance of these units is summarised in Table 1. Calculations have covered those combinations of the three second stage and three third stage motors which have been considered feasible and practicable.

A study was also made to determine the optimum size of the second stage motor for Black Knight; the result is the present design of the Kestrel solid motor. This study considered two nominal payloads of 110 lb and 250 lb in conjunction with a third stage velocity increment of 700 ft/sec and two different first stages, the Gamma 301 with 21,600 lb weight thrust and the 54" version. It was concluded that a charge weight of near 1250 lb was best, in combination with the highest possible specific impulse and mass fraction.

2.4 Two stage vehicle, both stages firing sequentially upwards

A two stage vehicle in which both stages fire upwards has application as a sounding rocket. It is the best method of achieving high performance, but,

in the role of re-entry test vehicle, the high dispersion which may arise is undesirable. If the second stage is fired immediately after the first stage burn-out, followed by head separation, then the time of flight (between head separation and re-entry) is so long that only a few ft/sec separation velocity is necessary to yield a large separation between head and second stage at re-entry and the complete weight of the separation system is negligible. A limited number of calculations have been made and these show gains in performance over the present three stage system of up to 3000 ft/sec in re-entry velocity.

2.5 Two stage intermediate range ballistic missile (I.R.B.M.)

The four configurations considered above have the common objective of reaching a high vertical velocity; the next three differ in that a high horizontal velocity is the main objective so that the trajectory must follow a curved path.

An investigation was recently made into the possible use of Black Knight as an I.R.B.M. Two cases were considered, both based on the 54" Black Knight with a sea level thrust of 25,000 lb weight. The first assumes that HTP and kerosene were the propellants, with a vacuum specific impulse of 250 secs; the second that the specific impulse can be raised to 280 secs by using propellants such as NTO/UDMH. In both cases the second stage used the same propellants as the first stage. The maximum range of an 800 lb re-entry vehicle was found to be just over 1000 n. miles with the HTP/kerosene combination and 1500 n. miles with NTO/UDMH.

2.6 Satellite launching vehicle

2.6.1 36" vehicle, liquid hydrogen second stage, and solid propellant third stage

If it were decided to proceed with liquid hydrogen engine development in this country one of the essential stages in such development would be to test the engine at high altitude. Apart from Blue Streak the only suitable first stage available in this country for such testing would be Black Knight. Preliminary estimates of the performance of this combination soon made it evident that orbital speeds could be achieved, and so a very limited number of further calculations were done to investigate in general terms the possibility of using such a combination as a launcher of satellites.

This investigation was made before the 54" Black Knight became a possibility and a 3 ft vehicle was assumed fitted with the Gamma 301 engine with a sea level thrust of 21,600 lb weight. Preliminary calculations showed that because of the small size of Black Knight, unless a very low orbit of 100 miles was required it would not be possible to thrust continuously into orbit with a constant second stage thrust and obtain a finite payload. An efficient scheme would need two levels of thrust. It is easier however to burn the second stage for a short period at a high thrust, coast up to orbit height and then add the final velocity increment with a small solid fuelled third stage.

Calculations were made for such a three stage vehicle with the following assumptions:-

(a) First stage, a 36" diameter Black Knight fitted with a Gamma 301 engine giving a sea level thrust of 21,600 lb and S.I. of 214 secs.

(b) Total take-off weight maintained at 17,280 lb to give a 1·25g take-off acceleration.

(c) Second stage fitted with a liquid hydrogen motor with a vacuum specific impulse of 400 secs.

(d) Third stage solid motor with vacuum specific impulse of 250 secs.

(e) The orbit assumed was a 300 nautical mile circle.

Various second stage weights and motor thrusts have been used in order to determine the effect of such changes.

2.6.2 <u>54" Black Knight, Kestrel solid second stage and solid third stage</u>

Directly stemming from an investigation into the use of Black Knight as a launcher of hypersonic gliders, (described below) came the suggestion of a satellite launcher comprising the 54" Black Knight and solid second and third stages. Calculations have been made for one such combination as follows:-

(a) First stage - 54" Black Knight with Gamma 303 engine giving sea level thrust of 25,000 lb and sea level S.I. of 220 secs.

(b) Total take-off weight 20,000 lb to give a take-off acceleration of 1·25g.

(c) A Kestrel second stage with a vacuum specific impulse 270 secs and propellant weight of 1080 lb.

(d) Third stage solid motor with vacuum specific impulse 250 secs.

The results are only intended to give a general idea of what is possible, a fuller discussion of a Black Knight satellite launcher has been published as a separate report[1].

2.7 Launcher for a hypersonic glider

Black Knight has been proposed as a launcher of gliders at hypersonic speeds, partly as a study of hypersonic flight in general and also later to simulate the recovery of satellites. The problems that arise are similar to those present in using Black Knight in the role of satellite launcher but the lower trajectory required makes the heating and control problems worse.

The first requirement was to launch a glider at high altitude at Mach numbers up to 10, one particular case being horizontal flight at 180,000 ft altitude with velocity of 10,000 f.p.s.

The second case considered was the possibility of launchings at much higher velocities - at least 18,000 f.p.s., preferably higher - but at a greater altitude, viz. 300,000 ft, again in horizontal flight.

Preliminary calculations showed that the 36" diameter Black Knight with Gamma 301 engine had just sufficient performance to meet the first requirement. However, because the span of the glider, 4 ft at least, is greater than the vehicle diameter, the stability at the time of maximum aerodynamic loading is a major problem.

Calculations for the second case have taken the 54" Black Knight with 25,000 lb weight sea level thrust and the Kestrel second stage which is fired at first stage apogee. It is shown that 18,000 ft/sec can be reached with a 500 lb glider.

Aerodynamic heating is a problem in both cases, but it is more severe in the first case because of the shallower trajectory.

3 METHOD OF COMPUTATION AND PRESENTATION OF RESULTS

3.1 Near vertical trajectories

For this type of trajectory the velocity and height at burn-out may be used as a measure of performance, but a more convenient quantity is the re-entry velocity V_E at 200,000 ft. Above this height a vacuum ballistic trajectory can be assumed and below it, most re-entry effects become apparent. A further advantage of using V_E is the simplification of a single parameter instead of two. Most results for near vertical firings have therefore been given in terms of V_E or the changes in V_E by subsequent stages.

3.1.1 First stage performance

The equations of motion of the first stage during burning are given in Appendix 1 and the calculations were performed by integrating these equations on an electronic digital computer. The propellant flow rate is assumed to be constant and only the effect of exit pressure on thrust is taken into account. The assumed values of the basic constants are given in Table 1, page 34; the variation of drag coefficient C_A with Mach No. M, assuming the present 3 ft diameter vehicle is given in Fig.1.

The variation of V_{E1} with empty weight W_E for the Gamma 201 Black Knight is shown by curve I of Fig.2. All other results are concerned with the Gamma 301 vehicle.

The values of the velocity and height achieved at the end of first stage thrusting are given in nomogram form in Figs.3 and 4. These show the variation of first stage "all-burnt" velocity and height for different values of sea level thrust and specific impulse with varying take-off and all-burnt weight.

One interesting fact which emerges from these calculations is the measure of the loss of performance due to the atmosphere. If it is assumed that all the burning phase were to occur in vacuo, an analytical expression can be derived for the all-burnt velocity; numerical values derived from this expression have been compared with those from the computer programme. The difference between the two values represents the loss in velocity due to the presence of the atmosphere; the results are shown in Fig.5 and the variation with thrust and take-off weight are surprisingly small.

To obtain the velocity V_{E1} at re-entry it is necessary to take the vacuum ballistic trajectory into account. In vacuo with zero thrust the equations of motion can be integrated analytically to obtain algebraic expressions for velocity and time as functions of height. These are well-known and the relevant equations are given in Appendix 2. Certain results based on these equations are given in Tables 3 and 4; Table 3 gives the time to return to 200,000 ft as a function of all-burnt height and velocity; Table 4 gives V_E as a function of the same parameters. These tables are applicable to any near vertical ballistic trajectory in vacuo and the values are frequently required for a variety of purposes in connection with Black Knight. They are essential for certain range operations such as re-entry predictions and are repeatedly required for such purposes as the setting of timing mechanisms and during analysis. Another most useful application has been to determine the all-burnt velocity immediately following a trial when the time from all-burnt to re-entry is usually obtained by stop-watch. The estimate of all-burnt velocity determined in this way is quite accurate, the times being insensitive to changes in all-burnt height but very sensitive to changes in all-burnt velocity. Because of the wide application of these values they have been quoted in detail in the present Note. The variation of apogee height with V_E, assuming a climb angle of $86{\cdot}5^\circ$, is shown in Fig.6.

The remaining results on first stage performance do not add any extra data to that provided by the nomograms, but the effects of changing particular parameters is best illustrated by replotting the results in a variety of ways. For this purpose the re-entry velocity V_{E1} is the best variable to use.

Fig.7 shows how V_{E1} varies with empty weight W_e and propellant weight W_p for a constant sea level specific impulse I of 213 secs and for three values of thrust T. The information is given in the form of a 4 dimensional carpet, the three 3 dimensional carpets being spaced to facilitate interpolation for intermediate values of T. For a given W_e and T the highest performance is obtained for the lowest value of thrust weight ratio at take-off. In practice, to avoid possible difficulties which might arise with a very slow take-off, the lowest limit of thrust/weight ratio is $1{\cdot}25$; this case is illustrated in Fig.8 for the variables W_e, T and V_{E1} at a constant I of 213 secs.

The effect of changes in the specific impulse is shown in Fig.9 where the rate of change of the re-entry velocity V_{E1} with change of specific impulse is

shown as a function of the re-entry velocity achieved for a specific impulse of 213 secs. This empirical relationship can only be approximate since changes in other parameters will also affect dV_{E1}/dI, but the computations show that the maximum error will not be more than about 10 feet per second over the range of values of the specific impulse possible with the Gamma engine.

One characteristic of the Gamma engine is that, apart from any changes in specific impulse resulting from design changes, the specific impulse varies directly with thrust for any one design. When calculating particular cases it is necessary to allow for this interdependence of thrust and specific impulse. Fig.10 shows this relationship for the current design of the Gamma 301 engine; it is based on experimental results achieved by the engine manufacturers.

Fig.2 shows the variation of V_{E1} with W_e for a range of possible vehicles; apart from curve I, which is for the Gamma 201 engine, they all apply to the Gamma 301 engine. The dashed curve V is for a vehicle with a constant propellant weight W_p of 11,600 lb and a sea level thrust of 21,600 lb weight; this is the form of first stage that will be in use in the Black Knight firings for the next few years. The other curves all assume a thrust/weight ratio of 1·25 at take-off. Two curves of an enlarged Black Knight with a sea level thrust of 25,000 lb weight and specific impulse of 220 secs are shown, one assumes that the tanks are lengthened and the standard drag curve of Fig.1 applies, the other assumes 54" diameter tanks and scales the standard drag in proportion to the area. The 500 ft/sec difference between the curves is therefore the result of the extra drag but is certainly pessimistic.

3.1.2 Performance of two stage vehicles

(a) Upward firings

The equations of motion of the second stage during burning, when it is fired on ascent immediately after first stage burn-out, are the same as those for the first stage. It is true that we are now dealing with an unguided rocket but it is spin stabilized and the burning time is short so that errors are small. The results for upward firings shown in Fig.11 were therefore obtained by the same method as the first stage calculations.

At present there are no definite plans for firing the second stage upwards so generalised calculations were not justified. Certain specific cases have been taken and the results are shown in Fig.11, again using the re-entry velocity V_{E2} as the measure of performance. This shows the variation in velocity with payload, (by payload is meant the weight of the re-entry head or equivalent experimental capsule).

This method of firing in addition to giving high re-entry velocity also provides a very high altitude sounding rocket. The apogee heights achieved are also shown in Fig.11.

(b) Downward firings

For downward firings the method of calculation of performance is necessarily more complex and was again programmed for a digital computer. This programme was primarily intended to calculate detailed trajectory information for trial purposes and applies the vacuum trajectory theory of Appendix 2 to find the velocity co-ordinates at the desired second stage ignition heights. It then adds the velocity increment provided by the second stage, taking into account the orientation of the second stage during burning; this is assumed to be the same as at the end of first stage burning. Although this programme was used to calculate the published results, its elaborations are really unnecessary for calculations of performance only and the simplified approach given in Appendix 3 would be sufficiently accurate.

For these calculations the second stage ignition height was assumed to be 350,000 feet and the re-entry velocity (at 200,000 feet) calculated for a variety of first and second stage characteristics. From these results the net additional velocity ΔV_E was derived, i.e. the difference between the re-entry velocity achieved with a particular two stage vehicle and the re-entry velocity which would be achieved with the same vehicle but with second stage motor not firing.

The two important factors determining this velocity increase are:-

(a) the specific impulse of the second stage motor, and

(b) the ratio λ_T of the complete second stage weights before and after motor burning.

Other factors are the burning time of the second stage motor and the velocity at first stage burn-out. It was found however that the effect of changes in these two quantities were small and could be neglected over the range of likely values. This made it possible to present the results in one single figure involving the two important variables only. The results obtained are shown in Fig.12; the second stage burning time assumed was 10 secs and first stage all-burnt velocity 9500 feet/sec.

(c) Variation in re-entry velocity with variation in second stage ignition height and time

It is shown in Appendix 3 that when the second stage is fired downwards t secs after first stage all-burnt the re-entry velocity is given approximately by

$$V_{E2} = (2gt\,\Delta V_2)^{\frac{1}{2}} \qquad (1)$$

where ΔV_2 is the actual increment of velocity due to the second stage motor. (This must not be confused with ΔV_E above. The value ΔV_2 can be determined directly knowing the characteristics of the second stage motor by integrating $g\,T_2/W_2$ over the motor burning time, T_2 being the motor thrust and W_2 the second stage weight, both being functions of the burning time).

This approximation, which states that V_{E2} is independent of the first stage all-burnt velocity, can only apply when certain conditions defined in Appendix 3 are maintained. However this is so over a very wide range of first stage conditions. For example, accurate calculations have confirmed that for rounds using a Cuckoo II on a standard Black Knight the variation in V_{E2} is less than 1 ft/sec, for 1000 ft/sec change in the first stage all-burn velocity, when the ignition time is kept constant. This relationship then provides a simple method of maintaining a constant re-entry velocity for a series of rounds when the first stage performance may vary from round to round or when the first stage under performs due to some engino malfunction.

It follows from equation (1) that the maximum re-entry velocity V_{E2} is obtained for the greatest value of t, that is the second stage should be fired as late or as low as possible.

The loss in V_{E2} due to an increase of ignition height is obtained by differentiating equation (1) to give

$$\delta V_{E2} = \left[g\Delta V/V_{E2} \, (V_1 - gt \right] \delta H \ . \tag{2}$$

In a typical case $V_{E2} = 15{,}000$ ft/sec and $\Delta V = \frac{1}{2}V_{E2} = V_1$ the burn-out velocity of the first stage and $t = 500$ secs, thus

$$\delta V_{E2} = -0{\cdot}002 \ \delta H \tag{3}$$

or a loss of 200 ft/sec per hundred thousand feet increase in ignition height.

3.1.3 <u>Performance of three-stage vehicles</u>

(a) <u>Effect of providing head separation</u>

There are two aspects of calculations on the third stage performance. Firstly the ignition height to provide the necessary head separation at re-entry must be determined and then the resulting trajectories of head and second stage boost calculated. The ignition height required will of course be directly determined by the head separation velocity which can be achieved. Because of limitations imposed by the sabot system the maximum separation velocity can only be a few hundred feet per sec, which means the ignition height must be large (if the separation distance at re-entry between head and second stage is to be large). It follows from para 3.1.2 that unless there is an appreciable gain in velocity from the third stage the provision of a large separation distance between head and second stage must result in a reduction in re-entry velocity.

In practice there is usually a small net loss in velocity increment arising from the addition of the third stage, i.e. if a two stage vehicle with the second stage fired at 350,000 feet were replaced by a three stage vehicle with the same second stage ignition height, the actual re-entry velocity would be slightly less. This is because the second stage has to accelerate the extra weight of the sabot and third stage, and the velocity loss of about 500 ft/sec or more is not completely compensated for by the third stage increment of 400 ft/sec.

It is shown in Appendix 3 that equation (1) may be used to estimate three stage performance, except that ΔV_2 is replaced by $(\Delta V_2 + \Delta V_3)$ and as we have seen these are almost numerically equal. The ignition time t is determined by the desired ignition height and is given approximately by

$$t = 2V_o/g - 2t_s$$

where V_o is the first stage velocity at all-burnt and t_s is the time required for a separation of x ft to be achieved, i.e. $t_s = x/\Delta V_3$.

Now let t be the time of ignition if x ft of separation is desired and t_m be the time for ignition if maximum performance is required, i.e. the second stage will have finished thrusting at 200,000 feet giving no head separation before re-entry. Then t_m would correspond to ignition at 350,000 feet.

Therefore

$$t_m \simeq 2V_o/g$$

$$t = t_m - 2t_s \quad . \tag{4}$$

Hence if V_{E3} is the re-entry velocity for a separation of x ft and $V_{E3}(\max)$ the maximum possible re-entry velocity, it follows from equations (1) and (4) that

$$V_{E3} = (1 - 2t_s/t_m)^{\frac{1}{2}} \cdot V_{E3}(\max)$$

or approximately

$$V_{E3} = (1 - t_s/t_m)\, V_{E3}(\max) \; . \tag{5}$$

Thus if the third stage increment ΔV_3 is 400 ft/sec and x is 20,000 ft then t_s is 50 secs and in a typical oase t_m is 600 secs and V_{E3} (max) about 16,400 ft/sec, the third stage re-entry velocity will be 15,000 ft/sec or the penalty for a separation of 20,000 ft is 1,400 ft/sec.

(b) Estimating the performance of the general three stage case

General performance curves for two values of the first stage performance, V_{E1}, are given in Fig.13, (a) at 9500 ft/sec, (b) at 10,500 ft/sec. The curves are in the form of 4-dimensional carpets. ΔV_E, the change in re-entry velocity from second and third stages is given as a function of λ_T and I, of the second stages for three values of ΔV_3, viz 400, 500 and 700 ft/sec. The three carpets are separated so that results for intermediate values of ΔV_3 can be obtained by interpolation. Results for general values of V_{E1} can be obtained by linear extrapolation or interpolation between Figs.13(a) and 13(b); this has been found to be empirically valid between the expected range of V_{E1} of 8000 to 12,000 ft/sec.

These results were not obtained by application of equation (5) which is only useful for making rough estimates, but were produced on a digital computer using an extension of the method used for 2 stages and briefly described in para 3.1.2(b). In this programme the ignition height is found by an iterative trial and error process.

(c) The optimum size of second stage

A study has been made to determine the optimum size of Kestrel motor for nominal payloads of 110 lb and 250 lb in conjunction with a third stage velocity of 700 ft/sec, making due allowance for the effect of the second and third stage weights on first stage performance. The variation of V_{E3} for these two cases with μ and W_p is shown in Fig.14, μ being the mass fraction of the motor and W_p the charge weight. Results for two first stages are given, the present Black Knight with a Gamma 301 motor at 21,600 lb weight thrust and the 54" Black Knight with 25,000 lb weight thrust. The specific impulse of the second stage I is assumed to be 270 secs. It will be seen that the effect of changes in W_p are small and a value of 1250 lb is about the optimum in all four cases. The effect of changes in I are shown in Fig.15 for the same cases and a constant charge weight of 1250 lb. It will be seen that charge weight in itself is not critical and should be chosen so that I and μ are a maximum.

(d) Some special cases

A comparison between several three stage configurations is given in Fig.16, which shows the variation of re-entry velocity with payload. The combinations include the two types of first stage mentioned above and the second stages are the Cuckoo II and the Kestrel. The third stage is assumed to be IMP Xs, the number being varied with the payload weight so that the separation velocity is 400 ft/sec. The diagram well illustrates the effects of improvements in both first and second stages. It is worth noting that the Black Knight with Gamma 201 engine in combination with Cuckoo I second stage and 4 IMP X motors for the third stage gives a V_{E3} of 14,000 ft/sec with

110 lb head; there is an increase of 3500 ft/sec by changing to the Gamma 301 engine with the Cuckoo II and a further 4000 ft/sec from the 54" Black Knight with the Kestrel. A comparison with Fig.11 will show that a further 2500 ft/sec would be obtained by firing the second stage upwards at first stage all-burnt and doing away with the large third stage.

3.2 Two dimensional trajectories

Once a trajectory departs appreciably from the vertical, as it must to launch a satellite, for instance, the performance depends on the flight path as well as on the stage parameters. The equations of motion for a two dimensional trajectory are given in Appendix 1 and have been programmed for a digital computer, including facilities for multistage configurations. Aerodynamic effects are normally only important during first stage burning and the values assumed for C_A and $dC_n/d\alpha$ are given in Fig.1. The particular trajectory obtained depends on the variation of thrust angle ψ with time, or of vehicle incidence α with time. In general there is an infinite number of ψ programmes to achieve a particular mission but of these there will be an optimum, i.e. there is only one trajectory which achieves a particular mission with the maximum payload and does not violate restrictions placed on temperature rise due to aerodynamic heating, structural loads etc. It is these restrictions which make it more difficult to produce general performance curves of the form given for near vertical trajectories. The methods by which the ψ programmes have been found vary from trial and error to sophisticated calculations using variation techniques, and are not discussed in this paper.

3.2.1 Two stage ballistic missile

A two stage vehicle with liquid propellant in both stages was considered, the second stage to be ignited at first stage burn-out. The 54" Black Knight was taken as the first stage with a sea level thrust of 25,000 lb weight and a vacuum S.I. of 250. The second stage was assumed to have the same S.I. but a range of initial weights and thrusts were tried in order to find the best combination. The dry weights were calculated from the following empirical formulae which are based on past experience both with Black Knight and Blue Streak, W_p is the propellant weight and T the thrust.

EMPTY WEIGHT OF FIRST STAGE $\quad W_E = 550 + 0{\cdot}017\, W_p + 0{\cdot}032\, T \qquad (6)$

EMPTY WEIGHT OF SECOND STAGE $\quad W_E = 280 + 0{\cdot}033\, W_p + 0{\cdot}031\, T \qquad (7)$

These expressions include the weight of guidance equipment, which is mainly carried in the second stage.

The thrust angle programme was found by trial and error, the best was a vertical rise for the first twenty seconds followed by a steady turnover at 0·55 deg/sec until the thrust angle was 40° to the horizontal (20, 0·55, 40) in short hand.

The calculations showed that range achieved, for a particular re-entry vehicle weight, was not very sensitive either to the second stage thrust and weight or turnover programme. It was found that a second stage weight of 3500 lb at a thrust of 6000 lb was about the optimum. Fig.17 shows how range varies with re-entry vehicle weight. A weight of 800 lb is about the minimum useful one and it will be seen that the HTP/KERO propellants (Vacuum SI = 250) would achieve a little less than 1100 nautical miles. Calculations were repeated with a Vacuum SI of 280, as could be obtained with UDMH/NTO, and it will be seen from Fig.16 that the range is then increased to 1500 n.miles.

3.2.2 Satellite launching vehicle

(a) 36" vehicle with liquid hydrogen second stage and solid propellant third stage

It must be emphasised that the purpose of the calculations described here was to get some idea of the possible capabilities of Black Knight in this context. Weight estimates of the second and third stages were difficult mainly because of the uncertainty of the weights of additional guidance and control equipment. More detailed calculations would be required if the suggestion of using Black Knight for this purpose were accepted.

Calculations were made for a three stage vehicle. The first stage is the standard Black Knight powered by the Gamma 301 engine set at a sea level thrust of 21,600 lb weight but carrying more propellant in lengthened tanks to give a thrust weight ratio at lift-off of 1·25. The second stage uses liquid oxygen and hydrogen and the third is a small but efficient solid motor.

The flight plan is very simple. After an initial vertical rise for 20 secs, the first stage is turned over during burning at a constant rate of 0·7° per sec until the thrust angle is 40° to the horizontal (20, 0·7, 40) to give a final climb angle of between 30 and 40 degrees, similar to the IRBM trajectory. The liquid hydrogen stage then follows a gravity turn, that is to say, the incidence α is kept at zero. Second stage burning terminates when the resulting apogee of the ballistic trajectory would be at the desired orbit height. When the third stage reaches this apogee it is ignited and adds the difference ΔV_3 between apogee velocity and orbital velocity at this height.

The following procedure is necessary in these calculations. Firstly a total second stage initial weight W_2 is chosen and the difference between this and the total take-off weight of 17,280 lb gives the first stage full weight. Using equation (6) of para. 3.2.1 W_p the fuel weight and W_E the empty first stage weight is calculated. This then enables the trajectory during first stage burning to be calculated for the flight plan described.

The next step is to calculate the trajectory during second stage burning using the initial weight already selected and assuming a value for the motor thrust, the SI being 400 secs. In addition to calculating the trajectory the computer programme calculates when to shut down the motor so that the resulting ballistic or coasting trajectory will have the desired apogee height. This is

also the orbital height required. The combined weight of the empty second stage and the full third stage at the end of second stage burning is therefore determined; at the same time the velocity at apogee is also calculated.

The third step is to determine what proportion of this weight is empty second stage weight W_{2E}. This includes all the electrical equipment, viz. guidance, control, telemetry etc. necessary as part of the second stage. Lack of experience of tank design and structure necessary for liquid hydrogen makes it difficult to arrive at any reliable estimates for tank weight and hence empty second stage weight. Because of this difficulty two arbitrary weights have been assumed viz. 450 lb and 750 lb.

Selecting one of these weights then enables the weight of the third stage to be calculated. The remaining step is to determine the empty weight of the third stage when the appropriate velocity increment has been achieved. For this it is assumed that the SI of the motor is 250 secs and that the velocity vector is horizontal during the thrusting period, so that the empty weight, which is the weight put into orbit is given by

$$W = (W_2 - W_{2E}) \exp \left(- \Delta V_3/gI_3\right) \quad . \tag{8}$$

Fig.18 shows the effect of second stage thrust and total weight on the weight that may be placed in a 300 n.m. orbit for two values of empty weight of 450 and 750 lb. The total weight has been varied between 2,500 and 4,000 lb, and the thrust from 4000 to 10,000 lb weight, the higher thrusts going with the higher weights.

Although the results suggest that it pays to have a large liquid hydrogen stage, weights above 3000 lb are not practicable because a 3 ft diameter vehicle would be much too long, moreover a larger second stage would have an empty weight of more than 750 lb. The general conclusion is that a 3000 lb second stage with 6000 lb weight thrust is about the optimum and an orbit weight of a little more than 100 lb could be expected.

(b) <u>54" vehicle with "Kestrel" solid second stage and solid third stage</u>

The calculations in connection with this configuration were of a purely exploratory nature and are merely an extension of the work done on the hypersonic glider. This case differs from that of the previous satellite case in that the first and second stages are actual vehicles being designed for other purposes. Thus the first stage comprises a 54" Black Knight with a Gamma 303 engine take-off thrust of 25,000 lb weight and take-off weight of 20,000 lb. The second stage is the Kestrel, a solid motor with charge weight 1080 lb and mass fraction 0·93. The calculation for the glider shows that a velocity of 18000 ft/sec can be achieved with a 500 lb glider at an altitude of 50 nautical miles. If this 500 lb included a solid third stage motor it could be used to add the remaining 8000 ft/sec to achieve orbital speed at 50 n.miles. Applying equation (8) with I_3 - 250 secs and $(W_2 - W_{2E})$ = 500 lb the orbit weight W is found to be 180 lb. Fifty miles is too low for a satellite orbit, much less than 180 lb could be placed in a 300 mile orbit and would be little more than the motor case.

It has only been possible to give a very brief outline of the satellite launching possibilities of Black Knight but a much more detailed account has been published separately[7]. The general conclusion was that Black Knight in its present form is too small to launch satellites with payloads greater than a few tens of pounds, unless a liquid hydrogen stage is used. It would be necessary to double the size of the present vehicle if payloads of a few hundred pounds are required.

3.2.3 Launcher for a hypersonic glider

Calculations for the first requirement of a horizontal velocity of 10,000 ft/sec at 180,000 ft using the present Black Knight with a sea level thrust of 19,000 lb weight, were made before sophisticated computing techniques had been developed, Only two glider weights were therefore considered and the trajectory obtained by a trial and error method. Velocities of 10,500 ft/sec and 8700 ft/sec were obtained for glider weights of 500 lb and 1000 lb respectively.

The second requirement was to launch a glider horizontally at 180,000 ft/sec at an altitude of 300,000 ft. Calculations were made firstly with the 54" Black Knight on its own, and then with Cuckoo II or Kestrel as second stages.

The first stage lifts off with a 1·25 thrust weight ratio and a thrust of 25,000 lb weight and follows a vertical trajectory for 20 seconds. A small change $\Delta\theta$ is then made in the climb angle and a gravity turn or zero incidence trajectory is followed to all-burnt. For a given all-burnt weight a value of $\Delta\theta$ can be found so that the first stage apogee is 300,000 ft. This was done by an iterative trial and error process on a digital computer. Second stage calculations were also simplified by assuming that the velocity increment is added impulsively at first stage apogee.

The performance obtained with this type of trajectory is shown in Fig.21 for the 54" Black Knight with and without a second stage and gives the velocity obtained at 300,000 ft as a function of payload. It will be seen that 18,000 ft/sec can be reached with a 500 lb payload on a Kestrel second stage. In making these calculations structure weights that apply to Black Knight in the re-entry test vehicle role have been used and the quoted payload is glider weight. A modified control system would be required in the first stage and it is not clear at present whether or not this would mean a weight increase. If the second stage is spin established as at present, it may prove necessary to "kill" the spin before separating the glider. If so additional weight would be required which would reduce performance.

As a typical example, fuller details of a trajectory for which the first stage has an apogee velocity of 10,000 ft/sec is given in Table 2. It probably represents the worst aerodynamic case and could be usefully the basis of further studies on heating and control problems.

SYMBOLS

A_e	sq ft	exit area of engines
C_A		axial force coefficient
C_N		normal force coefficient
D	lb wt	drag force
g_o	ft per sec^2	acceleration due to gravity at sea level
h	ft	altitude
I	lb wt/lb per sec (secs)	specific impulse
M		Mach No.
m	slugs	mass
N	lb weight	normal force
P and P_o	lb per sq ft	pressure and sea level pressure
q	lb per sq ft	dynamic pressure ($\frac{1}{2}\rho V^2$)
r	ft	distance from oentre of earth
r_o	ft	radius of earth
S	sq ft	cross sectional area of missile
t	secs	time
T	lb wt	thrust
V	ft per sec	velocity
V_E	ft per sec	equivalent velocity at 200,000 ft
W	lb	weight
$\dot{W}$	lb per sec	flow rate of propellant
W_o	lb	weight at launch
W_e	lb	weight of empty stage
W_p	lb	weight of propellant
α	deg	incidence
γ	deg	latitude

SYMBOLS (Contd.)

δ	deg	direction of line of fire
θ	deg	flight path angle relative to local horizontal
ϕ	deg	range round earth
ψ	deg	thrust angle relative to horizontal at launch
μ		mass fraction of rocket
ρ	lb per cubic ft	density of air
ω	rad per sec	angular velocity
Ω	deg per sec	rate of rotation of earth
λ_T		ratio of initial and final weights of stage
θ_s	deg	flight path angle in inertial coordinates
ΔV	ft per sec	velocity increment

suffixes 1, 2, 3 etc. refer to stage number.

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ATTACHED:

Appendices 1-3
Tables 1 to 4 (Tables 3 and 4 Neg.Nos. 165,635-165,644
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APPENDIX 1

KINEMATICS OF THE FIRST STAGE OF BLACK KNIGHT IN THE ATMOSPHERE

GENERAL EQUATIONS OF MOTION OF A MISSILE OVER A FIXED EARTH IN TWO DIMENSIONS

Consider a particle moving over a spherical but non-rotating earth subject to the forces shown in Fig.20a. The most convenient equations are obtained by resolving the forces tangentially and normal to the trajectory.

$$\text{Normal forces} = m V \omega$$

where ω is the rate at which the tangent to the trajectory is turning.

Therefore

$$\omega = \dot{\theta} - \dot{\phi}$$

where θ is climb angle of the tangent relative to the local horizontal and $\dot{\phi}$ is the rate of change of angular range round the earth or rate of rotation of local horizontal i.e. $\dot{\phi} = V \cos \theta/r$ where r is distance from the centre of the earth.

Therefore

$$\left.\begin{aligned} \text{normal forces} &= mV [\dot{\theta} - V \cos \theta/r] \\ \text{tangential forces} &= m\dot{V} \ . \end{aligned}\right\} \qquad (1)$$

If α is angle of incidence of the body and the thrust relative to the trajectory we have

$$mV [\dot{\theta} - V \cos \theta/r] = [T - D] \sin \alpha + N \cos \alpha - m g \cos \theta$$

$$m\dot{V} = [T - D] \cos \alpha - N \sin \alpha - m g \sin \theta \ .$$

Therefore

$$\left.\begin{aligned} \dot{V} &= \frac{1}{m} [(T - D) \cos \alpha - N \sin \alpha] - g \sin \theta \\ V\dot{\theta} &= \frac{1}{m} [(T - D) \sin \alpha + N \cos \alpha] - g \cos \theta + V^2 \cos \theta/r \ . \end{aligned}\right\} \qquad (2)$$

Where m is the mass, T is the thrust D the drag and N the normal force. These are given by

where

$$\left.\begin{aligned}
m &= m_o - \dot{m}\,t \\
\dot{m} &= T_o/32{\cdot}17\ I_o \\
T &= T_o + (P_o - P)\ A_2 \\
D &= \tfrac{1}{2}\rho\, v^2\ S(C_A)_M \\
N &= \tfrac{1}{2}\rho\, v^2\ S(\partial C_N/\partial\alpha)_M\ \alpha \\
\text{and}\qquad \alpha &= \psi + \phi - \theta \\
g &= g_o\ r_o^2\ r^2\ .
\end{aligned}\right\} \tag{3}$$

By resolving along and normal to the local vertical we have

$$\dot{r} = V \sin\theta \tag{4}$$

$$\dot{\phi} = V \cos\theta/r\ . \tag{5}$$

Equations (2) - (5) have been programmed for a digital computer and used for the evaluation of performance for the satellite launcher and similar two-dimensional problems. A programme of ψ with time is found by a trial and error process which gives the desired end conditions. The variation of C_A and $\partial C_N/\partial\alpha$ with Mach No. M which have been used for all first stage calculations is shown in Fig.1.

Equation of motion for a "beam riding" missile

If the missile is flying along a fixed beam with its origin at the launcher and inclined at ψ to the horizontal then equations (2) - (5) may be greatly simplified, and even more so if the effect of incidence is taken as negligible.

We now have

$$\theta = \Psi + \phi$$

$$\dot{\theta} = \dot{\phi}$$

the rate of change of local horizontal.

Therefore

$$\dot{V} = \frac{1}{m}(T - D) - g \sin \theta \tag{6}$$

$$\dot{\theta} = \dot{\phi} = V \cos \theta / r \tag{7}$$

$$\dot{r} = V \sin \theta \tag{8}$$

These simpler equations have been used in a computer programme for all the calculations of first stage performance of Black Knight as test vehicle or sounding rocket.

APPENDIX 2

KINEMATICS OF BLACK KNIGHT AFTER ALL-BURNT IN VACUO

The various properties of ballistic orbits have been published extensively and only relevant results are presented here with no mathematical proof.

At the end of burning of the first stage the velocity V height h and climb angle θ are known in terms of coordinates fixed to the earth. The ballistic problem is simplified if we convert to inertial or space axes with origin at the centre of the earth. The relationship between the two systems of axes is illustrated in Fig.20b where the velocity and climb angle in space axes is represented by V_{S1} and θ_{S1}, and the direction by δ_{S1}. These are given by:

$$V_{S1}^2 = V^2 + 2V\Omega\, r_1 \cos\theta \cos\delta \cos\gamma_1 + \Omega^2 r_1^2 \cos^2\gamma_1 \tag{9}$$

$$\sin\theta_{S1} = V \sin\theta / V_{S1} \tag{10}$$

$$\tan\delta_{S1} = V\cos\theta \sin\delta / [(V\cos\theta\cos\delta + \Omega\, r \cos\gamma)] \;. \tag{11}$$

We require the velocity V_{S2}, the climb angle θ_{S2}, direction δ_{S2} and latitude γ_2 at a radius r_2 and this may be either side of the apogee.

From the conservation of energy and angular momentum of the orbit we have:

$$\tfrac{1}{2} V_{S1}^2 - g_o\, r_o^2 \big/ r_1 = \tfrac{1}{2} V_{S2}^2 - g_o\, r_o^2 \big/ r_2 \tag{12}$$

and

$$V_{S1}\, r_1 \cos\theta_1 = V_{S2}\, r_2 \cos\theta_2 \;. \tag{13}$$

V_{S2} and θ_{S2} may be obtained from equations (12) and (13) but it is more convenient to introduce a non dimensional energy parameter p

where

$$p_1 = \frac{r_1 V_{S1}^2}{g_o\, r_o^2} \;. \tag{14}$$

It then follows that:

$$\cos \theta_{S2} = \sqrt{\frac{p_1 \, r_1}{p_2 \, r_2}} \cdot \cos \theta_{S1}$$

$$V_{S2} = \sqrt{\frac{g_o \, r_o^2}{r_2} \, p_2}$$

where

$$p_2 = 2 - (2 - p_1) \frac{r_1}{r_2} \qquad (15).$$

To find γ_2 and δ_{S2} we need to know the range ϕ.

The range ϕ_1, from r_1 to the apogee is given by

$$\cos \phi_1 = 1 - \frac{p_1 \cos^2 \theta_1}{e} \qquad (16)$$

and ϕ_2 from the apogee to r_2 by

$$\cos \phi_2 = 1 - \frac{p_2 \cos^2 \theta_2}{e} \qquad (17)$$

where e is the eccentricity of the orbit and is given by:

$$e^2 = 1 - p_1 (2 - p_1) \cos^2 \theta_1 \, . \qquad (18)$$

Total range $\phi = \phi_1 \pm \phi_2$.

The motion in space is shown in Fig.21a. By spherical trigonometry γ_2 and δ_2 are given by:

$$\cos \gamma_2 = \sin \gamma_1 \cos \phi - \cos \gamma_1 \sin \phi \cos \delta_{S1} \qquad (19)$$

$$\cos \delta_{S2} = \frac{\sin \gamma_2 \cos \phi - \sin \gamma_1}{\cos \gamma_2 \sin \phi} \qquad (20)$$

V, θ, δ at r_2 in fixed earth coordinates are then given by:

$$V^2 = V_{S2}^2 + \Omega^2 r_2^2 \cos^2 \gamma_2 - 2 V_{S2} \Omega r_2 \cos \theta_{S2} \cos \delta_{S2} \cos \gamma_2 \tag{21}$$

$$\sin \theta = V_{S2} \sin \theta_2 / V \tag{22}$$

$$\tan \delta = V_{S2} \cos \theta_{S2} \sin \delta S_2 \Big/ (V_{S2} \cos \theta_{S2} \cos \delta_{S1} + \Omega r_2 \cos \gamma_2) . \tag{23}$$

The performance parameter V_E is the velocity obtained from Equation (21) when $r_2 = r_o + 200{,}000$ and is on the far side of the apogee, see Fig.21b.

V_E as a function of V and h at first stage all-burnt is given in Table 4 and it is assumed θ is $86{\cdot}5^\circ$ at all-burnt.

Other useful results are:

the apogee radius given by

$$r_A = (1 + e) a \tag{24}$$

where

$$a = r_1/(2 - p_1).$$

Apogee height $(r_A - r_o)$ is plotted as a function of V_E in Fig.6.

The time t_1 from apogee to r_1 is given by

$$t_1 = \left[\frac{a^3}{g_o r_o^2}\right]^{\frac{1}{2}} \left[\sqrt{e^2 - (1 - p_1)^2} - \cos^{-1}\left(\frac{1 - p_1}{e}\right)\right] \tag{25}$$

and t_2 from apogee to r_2 by

$$t_2 = \left[\frac{a^3}{g_o r_o^2}\right]^{\frac{1}{2}} \left[\sqrt{e^2 - (1 - p_1)^2} + \cos^{-1}\left(\frac{1 - p_1}{e}\right)\right] . \tag{26}$$

The total time $t = t_1 \pm t_2$.

Time from all-burnt to return to 200,000 ft is given as a function of all-burnt height and velocity in Table 3.

The above equations, with additions for presenting information in convenient coordinates, have been programmed for a digital computer and used for producing the results in this paper. A list of basic assumptions will be found in Table 1.

APPENDIX 3

KINEMATICS OF THE SECOND AND THIRD STAGES OF BLACK KNIGHT IN VACUO

The trajectory of the second and third stages during burning in vacuo must in general be considered as a problem in three dimensions. This is because the rocket is no longer controlled to fly in one plane along a particular flight path fixed to the ground. The theory is complex and will not be given here, but it has been programmed for a digital computer and used in the computation of results for this note. The effects of thrust misalignment and the gyration of the rocket were ignored and it was assumed that the second stage was spin stabilized in the direction of the guidance beam, (i.e. 86·5° elevation and 343° azimuth).

The burning time of the third stage is much less than one second and its velocity increment was therefore treated as instantaneous.

A greatly simplified theory for the second and third stage performance will now be given as it is useful for making quick estimates.

Performance of a three stage vehicle

Let ΔV_2 be the increment given by the second stage in time t_b.

Let ΔV_3 be the instantaneous increment in velocity given by the third stage at all-burnt of the second stage. Let ΔH_2 be the increment in distance covered by the second stage during burning.

Then if λ_2, λ_3 and I_2, I_3 are the weight ratios and specific impulses of the second and third stage motors

$$\left.\begin{aligned} \Delta V_2 &= g_o\, I_2 \log \lambda_2 \\ \Delta H_2 &= g_o\, I_2\, t_b \left[1 - \frac{\log \lambda_2}{\lambda_2 - 1}\right] \\ \Delta V_3 &= g_o\, I_3 \log \lambda_3 \quad . \end{aligned}\right\} \qquad (27)$$

Let the second stage be ignited t seconds after first stage all-burnt and be fired downwards. Let all burnt velocity and height of the first stage be h_o and V_o. Then height h_1, t seconds later is given by:

$$h_1 = h_o + V_o\, t - \tfrac{1}{2} g\, t^2 \qquad (28)$$

assuming that g is constant. The velocity is given by:

$$V_1 = g\,t - V_o \,. \tag{29}$$

Let V_2 and h_2 be the velocity and height at second stage all-burnt.

Then

$$V_2 = g\,t - V_o + \Delta V_2 + g\,t_b + \Delta V_3 \tag{30}$$

$$h_2 = h_o + V_o t - \tfrac{1}{2} g t^2 - g(t - V_o/g)\, t_b - \Delta H_2 \,. \tag{31}$$

Then velocity at height h_3 is given by:

$$V_3^2 = (\Delta V_2 + \Delta V_3 - V_o)^2 + 2g\,[(\Delta V_2 + \Delta V_3)\,(t + t_b) - \Delta H_2 + h_o - h_3] \,. \tag{32}$$

From this we deduce that for a two stage vehicle the velocity V_{E2} at 200,000 ft is given by:

$$V_{E2} = \sqrt{(\Delta V_2 - V_o)^2 + 2g\,[\Delta V_2(t + t_b) - \Delta H_2 + h_o - 200{,}000]} \,. \tag{33}$$

For a three stage vehicle if separation is required at h_3 then the time from all-burnt of the third stage to h_3 must be t_s where t_s is given by:

$$t_s = x/\Delta V_3 \tag{34}$$

where x is the separation required.

Therefore V_3 is also given by:

$$V_3 = V_2 + g\,t_s = (\Delta V_2 - V_o) + g\,(t + t_b + t_s) \,. \tag{35}$$

Combining equations (32) and (35) to eliminate V_3 an equation is obtained for t the ignition time:

$$t = \frac{V_o}{g} - t_s - t_b + \frac{1}{g}\sqrt{V_o^2 - 2\,g\left[(\Delta V_2 + \Delta V_3)\,t_s + \Delta H_2 - h_o + h_3\right]}$$

... (36)

Note that the first term in equation (36) is the first stage time to apogee. Having obtained t from equation (36) V_{E3} can be deduced from equation (32). Provided a suitable mean value of g is chosen (28 ft/sec^2 approx) then the results given by this simple theory are in good agreement with those obtained by the full theory and which are published herein. However the main use of these simple expressions is an aid to understanding the physical properties of multistage vehicles.

For two stage vehicles it can be deduced from equation (33).

(a) For maximum V_{E2} on an upward firing (ΔV_2 is - ve) t should be zero.

(b) For maximum V_{E2} on a downward firing t should be as great as possible i.e. ignition height should be as low as possible.

(c) If ΔV_2 is approximately equal to V_o

then

$$V_{E2} \simeq \sqrt{2\, g\, \Delta V_2 (t + t_b)} \tag{38}$$

because $(-\Delta H_2 + h_o - 200{,}000)$ is usually about zero.

In which case V_{E2} is independent of V_o. Accurate calculations have confirmed that, for rounds using the Cuckoo II on a standard Black Knight, the variation in V_{E2} is less than 1 ft/sec for a 1000 ft/sec change in V_o, when ignition time is constant.

For three stage vehicles we may combine equations (32) and (36), making h_3 = 200,000 ft to obtain V_{E3} as a function of separation time t_s. Once again we may approximate and obtain a similar relation to equation (38)

$$V_{E3} \simeq \sqrt{2g\, (\Delta V_2 + \Delta V_3) \left(2 \frac{V_o}{g} - 2\, t_s\right)} \tag{39}$$

TABLE 1

List of basic assumptions

Radius of earth	=	$2{\cdot}09078 \times 10^{7}$ ft
Rate of rotation of earth	=	$7{\cdot}29212 \times 10^{-5}$ rad/sec
Latitude of launcher	=	31° 02' 27" S
Acceleration due to gravity at sea level	=	$32{\cdot}2$ ft/sec^2

Direction of guidance beam (For re-entry test vehicle)

Elevation = 86·5 deg
Azimuth = 343·0 deg

This is the normal direction for two stage rounds.

Nominal weights of first stage less fuel + payload (Re-entry test vehicle)

Fitted with 201 engine	=	1350 lb
Fitted with 301 engine	=	1380 lb
Fitted $4\frac{1}{2}$ ft diameter tanks and 301 engine	=	1480 lb

Nominal propellant capacity of first stage

Fitted with 3 ft diameter tanks	=	11,600 lb
Fitted with $4\frac{1}{2}$ ft diameter tanks	=	15,600 lb

Nominal weight of second stages less payload (Re-entry test vehicle)

Fitted with Cuckoo I	=	575 lb
Fitted with Cuckoo II	=	540 lb
Fitted with Kestrel	=	1225 lb

N.B. In addition to payload weight a further 45 lb must be added to obtain complete weight on first stage.

Nominal specification for second stage rockets

	Total weight	Charge weight	$\underline{I}$ (Vac)
Cuckoo I	530 lb	400 lb	222
Cuckoo II	494 lb	421 lb	250
Kestrel	1164 lb	1080 lb	271

TABLE 1 (Contd.)

Nominal specification for complete third stage less the re-entry head

		Velocity with 110 lb head	Total weight	Charge weight	I (Vac)
(a)	26 IMP 6	400 ft/sec	46 lb	8·5 lb	202
(b)	4 IMP 10	400 ft/sec	35 lb	6·8 lb	265
(c)	10 IMP 10	700 ft/sec	53 lb	13·6 lb	265

Note For (c) the charge weight is for 8 IMP 10; the remaining 2 IMP 10 being used to retard the sabot.

TABLE 2

Typical trajectory for launching hypersonic test vehicle

Time secs	Axial accln. g	Velocity ft/sec	Climb angle deg.	Height n. miles	q lb/sq ft
0	1·25	0	90·0	0	0
20	1·42	213	88·5	0·32	50
40	1·63	551	77·4	1·52	267
60	1·68	1061	61·1	3·95	614
80	1·98	1642	44·2	7·36	649
100	2·95	2842	30·6	11·60	575
120	4·25	4851	21·9	16·93	362
140	6·82	8090	16·6	23·64	152
149·44	9·30	10422	15·0	27·52	106

Range at flame out = 54 n. miles

Apogee - height = 50 n. miles Velocity = 10,000 ft/sec

Range at apogee = 210 n. miles (when second stage is fired)

Time at apogee = 245 secs

Time of ballistic impact = 385 secs or 140 secs from apogee

TABLE III

TIME (SECS) FROM ALL-BURNT TO RETURN TO 200000 FT.

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 200	205	210	215	220	225	230	235	240	245
7000	466.6	467.6	468.5	469.4	470.4	471.3	472.2	473.2	474.1	475.0
7100	474.0	474.9	475.8	476.8	477.7	478.6	479.6	480.5	481.4	482.3
7200	481.4	482.3	483.2	484.1	485.1	486.0	486.9	487.8	488.7	489.7
7300	488.8	489.7	490.6	491.6	492.5	493.4	494.3	495.2	496.1	497.0
7400	496.2	497.2	498.1	499.0	499.9	500.8	501.7	502.6	503.5	504.4
7500	503.7	504.6	505.6	506.5	507.4	508.3	509.2	510.1	511.0	511.9
7600	511.3	512.2	513.1	514.0	514.9	515.8	516.7	517.6	518.5	519.4
7700	518.8	519.7	520.6	521.5	522.4	523.3	524.2	525.1	526.0	526.9
7800	526.4	527.3	528.2	529.1	530.0	530.9	531.8	532.7	533.6	534.5
7900	534.0	534.9	535.8	536.7	537.6	538.5	539.4	540.3	541.2	542.0
8000	541.7	542.6	543.5	544.4	545.3	546.2	547.0	547.9	548.8	549.7
8100	549.4	550.3	551.2	552.1	553.0	553.8	554.7	555.6	556.5	557.4
8200	557.2	558.0	558.9	559.8	560.7	561.6	562.4	563.3	564.2	565.1
8300	564.9	565.8	566.7	567.6	568.5	569.3	570.2	571.1	572.0	572.8
8400	572.8	573.6	574.5	575.4	576.3	577.1	578.0	578.9	579.7	580.6
8500	580.6	581.5	582.4	583.2	584.1	585.0	585.8	586.7	587.6	588.4
8600	588.5	589.4	590.2	591.1	592.0	592.9	593.7	594.6	595.5	596.3
8700	596.5	597.3	598.2	599.1	599.9	600.8	601.7	602.5	603.4	604.2
8800	604.4	605.3	606.2	607.0	607.9	608.8	609.6	610.5	611.3	612.2
8900	612.5	613.3	614.2	615.0	615.9	616.8	617.6	618.5	619.3	620.2
9000	620.5	621.4	622.2	623.1	624.0	624.8	625.7	626.5	627.4	628.3
9100	628.6	629.5	630.4	631.2	632.1	632.9	633.8	634.6	635.5	636.4
9200	636.8	637.6	638.5	639.4	640.2	641.1	641.9	642.8	643.6	644.5
9300	645.0	645.8	646.7	647.6	648.4	649.3	650.1	651.0	651.8	652.7
9400	653.2	654.1	655.0	655.8	656.7	657.5	658.4	659.2	660.1	660.9
9500	661.5	662.4	663.2	664.1	665.0	665.8	666.7	667.5	668.3	669.2
9600	669.9	670.7	671.6	672.4	673.3	674.1	675.0	675.8	676.7	677.5
9700	678.3	679.1	680.0	680.8	681.7	682.5	683.4	684.2	685.1	685.9
9800	686.7	687.6	688.4	689.3	690.1	691.0	691.8	692.7	693.5	694.4
9900	695.2	696.0	696.9	697.7	698.6	699.5	700.3	701.2	702.0	702.8
10000	703.7	704.6	705.4	706.3	707.1	708.0	708.8	709.7	710.5	711.4
10100	712.3	713.2	714.0	714.9	715.7	716.6	717.4	718.3	719.1	720.0
10200	721.0	721.8	722.7	723.5	724.4	725.2	726.1	726.9	727.8	728.6
10300	729.7	730.5	731.4	732.2	733.1	733.9	734.8	735.6	736.5	737.3
10400	738.4	739.3	740.1	741.0	741.8	742.7	743.5	744.4	745.2	746.1
10500	747.2	748.1	748.9	749.8	750.6	751.5	752.3	753.2	754.0	754.9
10600	756.1	756.9	757.8	758.6	759.5	760.4	761.2	762.1	762.9	763.8
10700	765.0	765.9	766.7	767.6	768.4	769.3	770.1	771.0	771.8	772.7
10800	774.0	774.9	775.7	776.6	777.4	778.3	779.1	780.0	780.8	781.7
10900	783.0	783.9	784.7	785.6	786.5	787.3	788.2	789.0	789.9	790.7
11000	792.1	793.0	793.9	794.7	795.6	796.4	797.3	798.1	799.0	799.8
11100	801.3	802.2	803.0	803.9	804.7	805.6	806.4	807.3	808.1	809.0
11200	810.5	811.4	812.2	813.1	814.0	814.8	815.7	816.5	817.4	818.2
11300	819.8	820.7	821.5	822.4	823.2	824.1	825.0	825.8	826.7	827.5
11400	829.2	830.0	830.9	831.7	832.6	833.5	834.3	835.2	836.0	836.9
11500	838.6	839.4	840.3	841.2	842.0	842.9	843.7	844.6	845.5	846.3
11600	848.0	848.9	849.8	850.6	851.5	852.4	853.2	854.1	855.0	855.8
11700	857.6	858.5	859.3	860.2	861.1	861.9	862.8	863.7	864.5	865.4
11800	867.2	868.1	869.0	869.8	870.7	871.6	872.4	873.3	874.2	875.0
11900	876.9	877.8	878.6	879.5	880.4	881.2	882.1	883.0	883.8	884.7
12000	886.7	887.5	888.4	889.3	890.2	891.0	891.9	892.8	893.6	894.5
12100	896.5	897.4	898.2	899.1	900.0	900.9	901.7	902.6	903.5	904.3
12200	906.4	907.3	908.1	909.0	909.9	910.8	911.6	912.5	913.4	914.3
12300	916.4	917.2	918.1	919.0	919.9	920.8	921.6	922.5	923.4	924.3
12400	926.4	927.3	928.2	929.1	929.9	930.8	931.7	932.6	933.5	934.4
12500	936.5	937.4	938.3	939.2	940.1	941.0	941.9	942.7	943.6	944.5
12600	946.8	947.6	948.5	949.4	950.3	951.2	952.1	953.0	953.8	954.7
12700	957.0	957.9	958.8	959.7	960.6	961.5	962.4	963.3	964.2	965.0
12800	967.4	968.3	969.2	970.1	971.0	971.9	972.8	973.6	974.5	975.4
12900	977.8	978.7	979.6	980.5	981.4	982.3	983.2	984.1	985.0	985.9
13000	988.4	989.3	990.2	991.1	992.0	992.9	993.8	994.7	995.6	996.5

TABLE III (Continued)

TIME (SECS) FROM ALL-BURNT TO RETURN TO 200000 FT.

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 250	255	260	265	270	275	280	285	290	295
7000	475.9	476.8	477.8	478.7	479.6	480.5	481.4	482.3	483.2	484.1
7100	483.2	484.1	485.1	486.0	486.9	487.8	488.7	489.6	490.5	491.4
7200	490.6	491.5	492.4	493.3	494.2	495.1	496.0	496.9	497.8	498.7
7300	497.9	498.8	499.7	500.6	501.5	502.4	503.3	504.2	505.1	506.0
7400	505.3	506.2	507.1	508.0	508.9	509.8	510.7	511.6	512.5	513.4
7500	512.8	513.7	514.6	515.5	516.4	517.2	518.1	519.0	519.9	520.8
7600	520.3	521.2	522.0	522.9	523.8	524.7	525.6	526.5	527.4	528.2
7700	527.8	528.7	529.6	530.4	531.3	532.2	533.1	534.0	534.8	535.7
7800	535.3	536.2	537.1	538.0	538.9	539.7	540.6	541.5	542.4	543.2
7900	542.9	543.8	544.7	545.6	546.4	547.3	548.2	549.1	549.9	550.8
8000	550.6	551.4	552.3	553.2	554.1	554.9	555.8	556.7	557.5	558.4
8100	558.2	559.1	560.0	560.8	561.7	562.6	563.5	564.3	565.2	566.0
8200	565.9	566.8	567.7	568.5	569.4	570.3	571.1	572.0	572.9	573.7
8300	573.7	574.6	575.4	576.3	577.2	578.0	578.9	579.7	580.6	581.5
8400	581.5	582.3	583.2	584.1	584.9	585.8	586.7	587.5	588.4	589.2
8500	589.3	590.2	591.0	591.9	592.7	593.6	594.5	595.3	596.2	597.0
8600	597.2	598.0	598.9	599.8	600.6	601.5	602.3	603.2	604.0	604.9
8700	605.1	606.0	606.8	607.7	608.5	609.4	610.2	611.1	611.9	612.8
8800	613.1	613.9	614.8	615.6	616.5	617.3	618.2	619.0	619.9	620.7
8900	621.1	621.9	622.8	623.6	624.5	625.3	626.2	627.0	627.9	628.7
9000	629.1	630.0	630.8	631.7	632.5	633.4	634.2	635.1	635.9	636.8
9100	637.2	638.1	638.9	639.8	640.6	641.5	642.3	643.1	644.0	644.8
9200	645.3	646.2	647.0	647.9	648.7	649.6	650.4	651.3	652.1	653.0
9300	653.5	654.4	655.2	656.1	656.9	657.8	658.6	659.5	660.3	661.2
9400	661.8	662.6	663.5	664.3	665.2	666.0	666.8	667.7	668.5	669.4
9500	670.0	670.9	671.8	672.6	673.4	674.3	675.1	676.0	676.8	677.7
9600	678.4	679.2	680.1	680.9	681.8	682.6	683.5	684.3	685.1	686.0
9700	686.8	687.6	688.5	689.3	690.2	691.0	691.8	692.7	693.5	694.4
9800	695.2	696.1	696.9	697.7	698.6	699.4	700.3	701.1	702.0	702.8
9900	703.7	704.5	705.4	706.2	707.1	707.9	708.8	709.6	710.4	711.3
10000	712.2	713.1	713.9	714.8	715.6	716.4	717.3	718.1	719.0	719.8
10100	720.8	721.7	722.5	723.4	724.2	725.0	725.9	726.7	727.6	728.4
10200	729.5	730.3	731.2	732.0	732.8	733.7	734.5	735.4	736.2	737.1
10300	738.2	739.0	739.9	740.7	741.6	742.4	743.2	744.1	744.9	745.8
10400	746.9	747.8	748.6	749.5	750.3	751.1	752.0	752.8	753.7	754.5
10500	755.7	756.6	757.4	758.3	759.1	760.0	760.8	761.7	762.5	763.3
10600	764.6	765.5	766.3	767.1	768.0	768.8	769.7	770.5	771.4	772.2
10700	773.5	774.4	775.2	776.1	776.9	777.8	778.6	779.5	780.3	781.2
10800	782.5	783.4	784.2	785.1	785.9	786.8	787.6	788.5	789.3	790.2
10900	791.6	792.4	793.3	794.1	795.0	795.8	796.7	797.5	798.4	799.2
11000	800.7	801.5	802.4	803.2	804.1	804.9	805.8	806.6	807.5	808.3
11100	809.9	810.7	811.6	812.4	813.3	814.1	815.0	815.8	816.7	817.5
11200	819.1	819.9	820.8	821.6	822.5	823.4	824.2	825.1	825.9	826.8
11300	828.4	829.2	830.1	831.0	831.8	832.7	833.5	834.4	835.2	836.1
11400	837.7	838.6	839.5	840.3	841.2	842.0	842.9	843.7	844.6	845.5
11500	847.2	848.0	848.9	849.8	850.6	851.5	852.3	853.2	854.0	854.9
11600	856.7	857.5	858.4	859.3	860.1	861.0	861.8	862.7	863.6	864.4
11700	866.2	867.1	868.0	868.8	869.7	870.6	871.4	872.3	873.1	874.0
11800	875.9	876.7	877.6	878.5	879.3	880.2	881.1	881.9	882.8	883.7
11900	885.6	886.5	887.3	888.2	889.1	889.9	890.8	891.7	892.5	893.4
12000	895.4	896.2	897.1	898.0	898.8	899.7	900.6	901.4	902.3	903.2
12100	905.2	906.1	907.0	907.8	908.7	909.6	910.5	911.3	912.2	913.1
12200	915.2	916.0	916.9	917.8	918.6	919.5	920.4	921.3	922.1	923.0
12300	925.1	926.0	926.9	927.8	928.7	929.5	930.4	931.3	932.2	933.0
12400	935.2	936.1	937.0	937.9	938.7	939.6	940.5	941.4	942.3	943.1
12500	945.4	946.3	947.2	948.0	948.9	949.8	950.7	951.6	952.4	953.3
12600	955.6	956.5	957.4	958.3	959.2	960.0	960.9	961.8	962.7	963.6
12700	965.9	966.8	967.7	968.6	969.5	970.4	971.3	972.1	973.0	973.9
12800	976.3	977.2	978.1	979.0	979.9	980.8	981.7	982.6	983.5	984.4
12900	986.8	987.7	988.6	989.5	990.4	991.3	992.2	993.1	994.0	994.9
13000	997.4	998.3	999.2	1000	1001	1002	1003	1004	1005	1005

TABLE III (Continued)

TIME (SECS) FROM ALL-BURNT TO RETURN TO 200000 FT.

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 300	305	310	315	320	325	330	335	340	345
7000	485.0	485.9	486.8	487.7	488.6	489.5	490.4	491.3	492.2	493.1
7100	492.3	493.2	494.1	495.0	495.9	496.8	497.6	498.5	499.4	500.3
7200	499.6	500.5	501.4	502.2	503.1	504.0	504.9	505.8	506.7	507.5
7300	506.9	507.8	508.7	509.6	510.5	511.3	512.2	513.1	514.0	514.8
7400	514.3	515.2	516.0	516.9	517.8	518.7	519.5	520.4	521.3	522.2
7500	521.7	522.5	523.4	524.3	525.2	526.1	526.9	527.8	528.7	529.5
7600	529.1	530.0	530.9	531.7	532.6	533.5	534.4	535.2	536.1	537.0
7700	536.6	537.5	538.3	539.2	540.1	540.9	541.8	542.7	543.5	544.4
7800	544.1	545.0	545.8	546.7	547.6	548.4	549.3	550.2	551.0	551.9
7900	551.7	552.5	553.4	554.3	555.1	556.0	556.9	557.7	558.6	559.4
8000	559.3	560.1	561.0	561.9	562.7	563.6	564.4	565.3	566.2	567.0
8100	566.9	567.8	568.6	569.5	570.3	571.2	572.1	572.9	573.8	574.6
8200	574.6	575.4	576.3	577.2	578.0	578.9	579.7	580.6	581.4	582.3
8300	582.3	583.2	584.0	584.9	585.7	586.6	587.4	588.3	589.1	590.0
8400	590.1	590.9	591.8	592.6	593.5	594.3	595.2	596.0	596.9	597.7
8500	597.9	598.7	599.6	600.4	601.3	602.1	603.0	603.8	604.7	605.5
8600	605.7	606.6	607.4	608.3	609.1	610.0	610.8	611.7	612.5	613.4
8700	613.6	614.5	615.3	616.2	617.0	617.9	618.7	619.6	620.4	621.2
8800	621.6	622.4	623.3	624.1	625.0	625.8	626.7	627.5	628.3	629.2
8900	629.6	630.4	631.3	632.1	633.0	633.8	634.6	635.5	636.3	637.1
9000	637.6	638.5	639.3	640.1	641.0	641.8	642.7	643.5	644.3	645.2
9100	645.7	646.5	647.4	648.2	649.1	649.9	650.7	651.6	652.4	653.2
9200	653.8	654.7	655.5	656.3	657.2	658.0	658.9	659.7	660.5	661.4
9300	662.0	662.8	663.7	664.5	665.4	666.2	667.0	667.9	668.7	669.5
9400	670.2	671.1	671.9	672.7	673.6	674.4	675.3	676.1	676.9	677.8
9500	678.5	679.3	680.2	681.0	681.9	682.7	683.5	684.4	685.2	686.0
9600	686.8	687.7	688.5	689.3	690.2	691.0	691.9	692.7	693.5	694.4
9700	695.2	696.0	696.9	697.7	698.6	699.4	700.2	701.1	701.9	702.7
9800	703.6	704.5	705.3	706.2	707.0	707.8	708.7	709.5	710.3	711.2
9900	712.1	713.0	713.8	714.6	715.5	716.3	717.2	718.0	718.8	719.7
10000	720.7	721.5	722.3	723.2	724.0	724.9	725.7	726.5	727.4	728.2
10100	729.3	730.1	730.9	731.8	732.6	733.4	734.3	735.1	736.0	736.8
10200	737.9	738.7	739.6	740.4	741.3	742.1	742.9	743.8	744.6	745.4
10300	746.6	747.4	748.3	749.1	750.0	750.8	751.6	752.5	753.3	754.2
10400	755.4	756.2	757.0	757.9	758.7	759.6	760.4	761.2	762.1	762.9
10500	764.2	765.0	765.9	766.7	767.5	768.4	769.2	770.1	770.9	771.7
10600	773.1	773.9	774.7	775.6	776.4	777.3	778.1	779.0	779.8	780.6
10700	782.0	782.8	783.7	784.5	785.4	786.2	787.1	787.9	788.7	789.6
10800	791.0	791.8	792.7	793.5	794.4	795.2	796.1	796.9	797.7	798.6
10900	800.1	800.9	801.7	802.6	803.4	804.3	805.1	806.0	806.8	807.7
11000	809.2	810.0	810.9	811.7	812.6	813.4	814.3	815.1	816.0	816.8
11100	818.4	819.2	820.1	820.9	821.8	822.6	823.5	824.3	825.1	826.0
11200	827.6	828.5	829.3	830.2	831.0	831.9	832.7	833.6	834.4	835.2
11300	836.9	837.8	838.6	839.5	840.3	841.2	842.0	842.9	843.7	844.6
11400	846.3	847.2	848.0	848.9	849.7	850.6	851.4	852.3	853.1	854.0
11500	855.8	856.6	857.5	858.3	859.2	860.0	860.9	861.7	862.6	863.4
11600	865.3	866.1	867.0	867.8	868.7	869.6	870.4	871.3	872.1	873.0
11700	874.9	875.7	876.6	877.4	878.3	879.2	880.0	880.9	881.7	882.6
11800	884.5	885.4	886.2	887.1	888.0	888.8	889.7	890.5	891.4	892.3
11900	894.2	895.1	896.0	896.8	897.7	898.6	899.4	900.3	901.2	902.0
12000	904.0	904.9	905.8	906.7	907.5	908.4	909.3	910.1	911.0	911.8
12100	913.9	914.8	915.7	916.5	917.4	918.3	919.1	920.0	920.9	921.7
12200	923.9	924.8	925.6	926.5	927.4	928.2	929.1	930.0	930.8	931.7
12300	933.9	934.8	935.7	936.5	937.4	938.3	939.2	940.0	940.9	941.8
12400	944.0	944.9	945.8	946.6	947.5	948.4	949.3	950.2	951.0	951.9
12500	954.2	955.1	956.0	956.8	957.7	958.6	959.5	960.4	961.2	962.1
12600	964.5	965.3	966.2	967.1	968.0	968.9	969.8	970.6	971.5	972.4
12700	974.8	975.7	976.6	977.5	978.3	979.2	980.1	981.0	981.9	982.8
12800	985.2	986.1	987.0	987.9	988.8	989.7	990.6	991.5	992.3	993.2
12900	995.8	996.6	997.5	998.4	999.3	1000	1001	1002	1003	1004
13000	1006	1007	1008	1009	1010	1011	1012	1013	1014	1014

TABLE III (Continued)

TIME (SECS) FROM ALL-BURNT TO RETURN TO 200000 FT.

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 350	355	360	365	370	375	380	385	390	395
7000	494.0	494.9	495.7	496.6	497.5	498.4	499.2	500.1	501.0	501.9
7100	501.2	502.1	502.9	503.8	504.7	505.6	506.4	507.3	508.2	509.0
7200	508.4	509.3	510.2	511.1	511.9	512.8	513.7	514.5	515.4	516.3
7300	515.7	516.6	517.5	518.3	519.2	520.1	520.9	521.8	522.7	523.5
7400	523.0	523.9	524.8	525.6	526.5	527.4	528.2	529.1	530.0	530.8
7500	530.4	531.3	532.1	533.0	533.9	534.7	535.6	536.4	537.3	538.1
7600	537.8	538.7	539.5	540.4	541.3	542.1	543.0	543.8	544.7	545.5
7700	545.3	546.1	547.0	547.8	548.7	549.5	550.4	551.2	552.1	553.0
7800	552.7	553.6	554.5	555.3	556.2	557.0	557.9	558.7	559.6	560.4
7900	560.3	561.1	562.0	562.8	563.7	564.5	565.4	566.2	567.1	567.9
8000	567.9	568.7	569.6	570.4	571.3	572.1	573.0	573.8	574.6	575.5
8100	575.5	576.3	577.2	578.0	578.9	579.7	580.5	581.4	582.2	583.1
8200	583.1	584.0	584.8	585.7	586.5	587.4	588.2	589.0	589.9	590.7
8300	590.8	591.7	592.5	593.4	594.2	595.0	595.9	596.7	597.6	598.4
8400	598.6	599.4	600.3	601.1	601.9	602.8	603.6	604.5	605.3	606.1
8500	606.4	607.2	608.0	608.9	609.7	610.6	611.4	612.2	613.1	613.9
8600	614.2	615.0	615.9	616.7	617.6	618.4	619.2	620.1	620.9	621.7
8700	622.1	622.9	623.8	624.6	625.4	626.3	627.1	627.9	628.8	629.6
8800	630.0	630.8	631.7	632.5	633.4	634.2	635.0	635.9	636.7	637.5
8900	638.0	638.8	639.7	640.5	641.3	642.2	643.0	643.8	644.7	645.5
9000	646.0	646.9	647.7	648.5	649.4	650.2	651.0	651.9	652.7	653.5
9100	654.1	654.9	655.8	656.6	657.4	658.3	659.1	659.9	660.7	661.6
9200	662.2	663.0	663.9	664.7	665.5	666.4	667.2	668.0	668.9	669.7
9300	670.4	671.2	672.0	672.9	673.7	674.5	675.4	676.2	677.0	677.9
9400	678.6	679.4	680.3	681.1	681.9	682.8	683.6	684.4	685.2	686.1
9500	686.9	687.7	688.5	689.4	690.2	691.0	691.9	692.7	693.5	694.4
9600	695.2	696.0	696.9	697.7	698.5	699.4	700.2	701.0	701.8	702.7
9700	703.6	704.4	705.2	706.1	706.9	707.7	708.6	709.4	710.2	711.1
9800	712.0	712.8	713.7	714.5	715.3	716.2	717.0	717.8	718.7	719.5
9900	720.5	721.3	722.2	723.0	723.8	724.7	725.5	726.3	727.1	728.0
10000	729.0	729.9	730.7	731.5	732.4	733.2	734.0	734.9	735.7	736.5
10100	737.6	738.5	739.3	740.1	741.0	741.8	742.6	743.5	744.3	745.1
10200	746.3	747.1	748.0	748.8	749.6	750.4	751.3	752.1	752.9	753.8
10300	755.0	755.8	756.7	757.5	758.3	759.2	760.0	760.8	761.7	762.5
10400	763.8	764.6	765.4	766.3	767.1	767.9	768.8	769.6	770.4	771.3
10500	772.6	773.4	774.3	775.1	775.9	776.8	777.6	778.4	779.3	780.1
10600	781.5	782.3	783.2	784.0	784.8	785.7	786.5	787.3	788.2	789.0
10700	790.4	791.3	792.1	792.9	793.8	794.6	795.5	796.3	797.1	798.0
10800	799.4	800.3	801.1	801.9	802.8	803.6	804.5	805.3	806.1	807.0
10900	808.5	809.3	810.2	811.0	811.9	812.7	813.5	814.4	815.2	816.1
11000	817.6	818.5	819.3	820.2	821.0	821.8	822.7	823.5	824.4	825.2
11100	826.8	827.7	828.5	829.4	830.2	831.1	831.9	832.7	833.6	834.4
11200	836.1	836.9	837.8	838.6	839.5	840.3	841.2	842.0	842.9	843.7
11300	845.4	846.3	847.1	848.0	848.8	849.7	850.5	851.4	852.2	853.1
11400	854.8	855.7	856.5	857.4	858.2	859.1	859.9	860.8	861.6	862.5
11500	864.3	865.1	866.0	866.9	867.7	868.6	869.4	870.3	871.1	872.0
11600	873.8	874.7	875.5	876.4	877.3	878.1	879.0	879.8	880.7	881.5
11700	883.4	884.3	885.2	886.0	886.9	887.7	888.6	889.4	890.3	891.1
11800	893.1	894.0	894.8	895.7	896.6	897.4	898.3	899.1	900.0	900.8
11900	902.9	903.7	904.6	905.5	906.3	907.2	908.0	908.9	909.8	910.6
12000	912.7	913.6	914.4	915.3	916.2	917.0	917.9	918.7	919.6	920.5
12100	922.6	923.5	924.3	925.2	926.1	926.9	927.8	928.7	929.5	930.4
12200	932.6	933.5	934.3	935.2	936.1	936.9	937.8	938.7	939.5	940.4
12300	942.6	943.5	944.4	945.3	946.1	947.0	947.9	948.7	949.6	950.5
12400	952.8	953.7	954.5	955.4	956.3	957.1	958.0	958.9	959.8	960.6
12500	963.0	963.9	964.7	965.6	966.5	967.4	968.3	969.1	970.0	970.9
12600	973.3	974.2	975.0	975.9	976.8	977.7	978.6	979.5	980.3	981.2
12700	983.7	984.6	985.4	986.3	987.2	988.1	989.0	989.9	990.7	991.6
12800	994.1	995.0	995.9	996.8	997.7	998.6	999.5	1000	1001	1002
12900	1005	1006	1006	1007	1008	1009	1010	1011	1012	1013
13000	1015	1016	1017	1018	1019	1020	1021	1022	1022	1023

TABLE III (Continued)

TIME (SECS) FROM ALL-BURNT TO RETURN TO 200000 FT.

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 400	405	410	415	420	425	430	435	440	445
7000	502.7	503.6	504.5	505.3	506.2	507.1	507.9	508.8	509.6	510.5
7100	509.9	510.8	511.6	512.5	513.3	514.2	515.1	515.9	516.8	517.6
7200	517.1	518.0	518.8	519.7	520.6	521.4	522.3	523.1	524.0	524.8
7300	524.4	525.2	526.1	526.9	527.8	528.7	529.5	530.4	531.2	532.0
7400	531.7	532.5	533.4	534.2	535.1	535.9	536.8	537.6	538.5	539.3
7500	539.0	539.9	540.7	541.6	542.4	543.3	544.1	544.9	545.8	546.6
7600	546.4	547.2	548.1	548.9	549.8	550.6	551.5	552.3	553.2	554.0
7700	553.8	554.6	555.5	556.3	557.2	558.0	558.9	559.7	560.5	561.4
7800	561.3	562.1	563.0	563.8	564.6	565.5	566.3	567.2	568.0	568.8
7900	568.8	569.6	570.5	571.3	572.1	573.0	573.8	574.7	575.5	576.3
8000	576.3	577.2	578.0	578.8	579.7	580.5	581.4	582.2	583.0	583.9
8100	583.9	584.8	585.6	586.4	587.3	588.1	588.9	589.8	590.6	591.4
8200	591.6	592.4	593.2	594.1	594.9	595.7	596.6	597.4	598.2	599.1
8300	599.2	600.1	600.9	601.7	602.6	603.4	604.2	605.1	605.9	606.7
8400	607.0	607.8	608.6	609.5	610.3	611.1	612.0	612.8	613.6	614.4
8500	614.7	615.6	616.4	617.2	618.1	618.9	619.7	620.6	621.4	622.2
8600	622.6	623.4	624.2	625.1	625.9	626.7	627.5	628.4	629.2	630.0
8700	630.4	631.3	632.1	632.9	633.8	634.6	635.4	636.2	637.1	637.9
8800	638.4	639.2	640.0	640.8	641.7	642.5	643.3	644.2	645.0	645.8
8900	646.3	647.2	648.0	648.8	649.6	650.5	651.3	652.1	652.9	653.8
9000	654.3	655.2	656.0	656.8	657.7	658.5	659.3	660.1	661.0	661.8
9100	662.4	663.2	664.1	664.9	665.7	666.5	667.4	668.2	669.0	669.8
9200	670.5	671.3	672.2	673.0	673.8	674.7	675.5	676.3	677.1	677.9
9300	678.7	679.5	680.3	681.2	682.0	682.8	683.6	684.5	685.3	686.1
9400	686.9	687.7	688.6	689.4	690.2	691.0	691.9	692.7	693.5	694.3
9500	695.2	696.0	696.8	697.7	698.5	699.3	700.1	701.0	701.8	702.6
9600	703.5	704.3	705.2	706.0	706.8	707.6	708.5	709.3	710.1	710.9
9700	711.9	712.7	713.5	714.4	715.2	716.0	716.8	717.7	718.5	719.3
9800	720.3	721.1	722.0	722.8	723.6	724.4	725.3	726.1	726.9	727.7
9900	728.8	729.6	730.5	731.3	732.1	732.9	733.8	734.6	735.4	736.2
10000	737.3	738.2	739.0	739.8	740.7	741.5	742.3	743.1	744.0	744.8
10100	746.0	746.8	747.6	748.4	749.3	750.1	750.9	751.7	752.6	753.4
10200	754.6	755.4	756.3	757.1	757.9	758.7	759.6	760.4	761.2	762.1
10300	763.3	764.2	765.0	765.8	766.6	767.5	768.3	769.1	770.0	770.8
10400	772.1	772.9	773.8	774.6	775.4	776.2	777.1	777.9	778.7	779.6
10500	780.9	781.8	782.6	783.4	784.3	785.1	785.9	786.8	787.6	788.4
10600	789.8	790.7	791.5	792.3	793.2	794.0	794.8	795.7	796.5	797.3
10700	798.8	799.6	800.5	801.3	802.1	803.0	803.8	804.6	805.5	806.3
10800	807.8	808.6	809.5	810.3	811.2	812.0	812.8	813.7	814.5	815.3
10900	816.9	817.7	818.6	819.4	820.2	821.1	821.9	822.8	823.6	824.4
11000	826.0	826.9	827.7	828.6	829.4	830.2	831.1	831.9	832.8	833.6
11100	835.3	836.1	837.0	837.8	838.6	839.5	840.3	841.2	842.0	842.8
11200	844.5	845.4	846.2	847.1	847.9	848.8	849.6	850.4	851.3	852.1
11300	853.9	854.7	855.6	856.4	857.3	858.1	859.0	859.8	860.7	861.5
11400	863.3	864.2	865.0	865.9	866.7	867.5	868.4	869.2	870.1	870.9
11500	872.8	873.7	874.5	875.4	876.2	877.0	877.9	878.7	879.6	880.4
11600	882.4	883.2	884.1	884.9	885.8	886.6	887.5	888.3	889.2	890.0
11700	892.0	892.9	893.7	894.6	895.4	896.3	897.1	898.0	898.8	899.7
11800	901.7	902.6	903.4	904.3	905.1	906.0	906.8	907.7	908.5	909.4
11900	911.5	912.3	913.2	914.1	914.9	915.8	916.6	917.5	918.3	919.2
12000	921.3	922.2	923.1	923.9	924.8	925.6	926.5	927.4	928.2	929.1
12100	931.3	932.1	933.0	933.9	934.7	935.6	936.4	937.3	938.2	939.0
12200	941.3	942.1	943.0	943.9	944.7	945.6	946.5	947.3	948.2	949.1
12300	951.3	952.2	953.1	954.0	954.8	955.7	956.6	957.4	958.3	959.2
12400	961.5	962.4	963.3	964.1	965.0	965.9	966.8	967.6	968.5	969.4
12500	971.8	972.6	973.5	974.4	975.3	976.1	977.0	977.9	978.8	979.6
12600	982.1	983.0	983.9	984.7	985.6	986.5	987.4	988.2	989.1	990.0
12700	992.5	993.4	994.3	995.2	996.0	996.9	997.8	998.7	999.6	1000
12800	1003	1004	1005	1006	1007	1007	1008	1009	1010	1011
12900	1014	1014	1015	1016	1017	1018	1019	1020	1021	1022
13000	1024	1025	1026	1027	1028	1029	1030	1031	1031	1032

TABLE IV

V_E (FT/SEC) AS A FUNCTION OF ALL-BURNT HEIGHT AND VELOCITY

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 200	205	210	215	220	225	230	235	240	245
7000	7000	7022	7045	7067	7089	7111	7133	7156	7178	7199
7100	7100	7122	7144	7166	7188	7210	7232	7253	7275	7297
7200	7200	7222	7243	7265	7287	7308	7330	7351	7373	7394
7300	7300	7321	7343	7364	7386	7407	7428	7449	7470	7491
7400	7400	7421	7442	7463	7485	7506	7526	7547	7568	7589
7500	7500	7521	7542	7563	7584	7604	7625	7645	7666	7686
7600	7600	7621	7641	7662	7682	7703	7723	7743	7764	7784
7700	7700	7720	7741	7761	7781	7801	7822	7842	7862	7882
7800	7800	7820	7840	7860	7880	7900	7920	7940	7960	7979
7900	7900	7920	7940	7960	7979	7999	8019	8038	8058	8077
8000	8000	8020	8039	8059	8078	8098	8117	8136	8156	8175
8100	8100	8119	8139	8158	8177	8197	8216	8235	8254	8273
8200	8200	8219	8238	8257	8276	8295	8314	8333	8352	8371
8300	8300	8319	8338	8357	8376	8394	8413	8432	8450	8469
8400	8400	8419	8437	8456	8475	8493	8512	8530	8549	8567
8500	8500	8518	8537	8555	8574	8592	8610	8629	8647	8665
8600	8600	8618	8636	8655	8673	8691	8709	8727	8745	8763
8700	8700	8718	8736	8754	8772	8790	8808	8826	8843	8861
8800	8800	8818	8836	8853	8871	8889	8907	8924	8942	8959
8900	8900	8918	8935	8953	8971	8988	9005	9023	9040	9058
9000	9000	9017	9035	9052	9070	9087	9104	9122	9139	9156
9100	9100	9117	9135	9152	9169	9186	9203	9220	9237	9254
9200	9200	9217	9234	9251	9268	9285	9302	9319	9336	9353
9300	9300	9317	9334	9351	9368	9384	9401	9418	9434	9451
9400	9400	9417	9433	9450	9467	9483	9500	9516	9533	9549
9500	9500	9517	9533	9550	9566	9582	9599	9615	9632	9648
9600	9600	9616	9633	9649	9665	9682	9698	9714	9730	9746
9700	9700	9716	9732	9749	9765	9781	9797	9813	9829	9845
9800	9800	9816	9832	9848	9864	9880	9896	9912	9928	9943
9900	9900	9916	9932	9948	9963	9979	9995	10011	10026	10042
10000	10000	10016	10031	10047	10063	10078	10094	10110	10125	10141
10100	10100	10116	10131	10147	10162	10178	10193	10208	10224	10239
10200	10200	10215	10231	10246	10262	10277	10292	10307	10323	10338
10300	10300	10315	10331	10346	10361	10376	10391	10406	10422	10437
10400	10400	10415	10430	10445	10460	10475	10490	10505	10520	10535
10500	10500	10515	10530	10545	10560	10575	10590	10604	10619	10634
10600	10600	10615	10630	10644	10659	10674	10689	10703	10718	10733
10700	10700	10715	10729	10744	10759	10773	10788	10802	10817	10831
10800	10800	10815	10829	10844	10858	10873	10887	10902	10916	10930
10900	10900	10914	10929	10943	10958	10972	10986	11001	11015	11029
11000	11000	11014	11029	11043	11057	11071	11086	11100	11114	11128
11100	11100	11114	11128	11142	11157	11171	11185	11199	11213	11227
11200	11200	11214	11228	11242	11256	11270	11284	11298	11312	11326
11300	11300	11314	11328	11342	11356	11369	11383	11397	11411	11425
11400	11400	11414	11428	11441	11455	11469	11483	11496	11510	11524
11500	11500	11514	11527	11541	11555	11568	11582	11595	11609	11622
11600	11600	11614	11627	11641	11654	11668	11681	11695	11708	11721
11700	11700	11713	11727	11740	11754	11767	11780	11794	11807	11820
11800	11800	11813	11827	11840	11853	11867	11880	11893	11906	11919
11900	11900	11913	11926	11940	11953	11966	11979	11992	12005	12018
12000	12000	12013	12026	12039	12052	12065	12078	12091	12104	12117
12100	12100	12113	12126	12139	12152	12165	12178	12191	12204	12216
12200	12200	12213	12226	12239	12252	12264	12277	12290	12303	12316
12300	12300	12313	12326	12338	12351	12364	12377	12389	12402	12415
12400	12400	12413	12425	12438	12451	12463	12476	12489	12501	12514
12500	12500	12513	12525	12538	12550	12563	12575	12588	12600	12613
12600	12600	12613	12625	12637	12650	12662	12675	12687	12700	12712
12700	12700	12712	12725	12737	12750	12762	12774	12786	12799	12811
12800	12800	12812	12825	12837	12849	12861	12874	12886	12898	12910
12900	12900	12912	12924	12937	12949	12961	12973	12985	12997	13009
13000	13000	13012	13024	13036	13048	13060	13072	13084	13097	13108

TABLE IV (Continued)

V_E (FT/SEC) AS A FUNCTION OF ALL-BURNT HEIGHT AND VELOCITY

VEL.	ALL-BURNT HEIGHT. THOUSANDS OF FEET.									
FT/SEC	250	255	260	265	270	275	280	285	290	295
7000	7221	7243	7264	7286	7307	7329	7350	7371	7392	7414
7100	7318	7339	7361	7382	7403	7424	7445	7466	7487	7508
7200	7415	7436	7457	7478	7499	7520	7541	7561	7582	7603
7300	7512	7533	7554	7575	7595	7616	7636	7657	7677	7697
7400	7609	7630	7651	7671	7691	7712	7732	7752	7772	7792
7500	7707	7727	7747	7768	7788	7808	7828	7848	7868	7887
7600	7804	7824	7844	7864	7884	7904	7924	7943	7963	7983
7700	7901	7921	7941	7961	7980	8000	8020	8039	8058	8078
7800	7999	8019	8038	8058	8077	8096	8116	8135	8154	8173
7900	8096	8116	8135	8154	8174	8193	8212	8231	8250	8269
8000	8194	8213	8232	8251	8270	8289	8308	8327	8346	8364
8100	8292	8311	8330	8348	8367	8386	8404	8423	8441	8460
8200	8389	8408	8427	8445	8464	8482	8501	8519	8537	8556
8300	8487	8506	8524	8543	8561	8579	8597	8615	8634	8652
8400	8585	8603	8622	8640	8658	8676	8694	8712	8730	8748
8500	8683	8701	8719	8737	8755	8773	8791	8808	8826	8844
8600	8781	8799	8817	8834	8852	8870	8887	8905	8922	8940
8700	8879	8897	8914	8932	8949	8967	8984	9001	9019	9036
8800	8977	8994	9012	9029	9046	9064	9081	9098	9115	9132
8900	9075	9092	9109	9127	9144	9161	9178	9195	9212	9229
9000	9173	9190	9207	9224	9241	9258	9275	9292	9309	9325
9100	9271	9288	9305	9322	9338	9355	9372	9389	9405	9422
9200	9369	9386	9403	9419	9436	9453	9469	9486	9502	9519
9300	9468	9484	9501	9517	9534	9550	9566	9583	9599	9615
9400	9566	9582	9599	9615	9631	9647	9664	9680	9696	9712
9500	9664	9680	9697	9713	9729	9745	9761	9777	9793	9809
9600	9762	9778	9795	9810	9826	9842	9858	9874	9890	9906
9700	9861	9877	9893	9908	9924	9940	9956	9971	9987	10003
9800	9959	9975	9991	10006	10022	10037	10053	10069	10084	10100
9900	10058	10073	10089	10104	10120	10135	10151	10166	10181	10197
10000	10156	10171	10187	10202	10218	10233	10248	10263	10279	10294
10100	10254	10270	10285	10300	10315	10331	10346	10361	10376	10391
10200	10353	10368	10383	10398	10413	10428	10443	10458	10473	10488
10300	10452	10467	10482	10496	10511	10526	10541	10556	10571	10585
10400	10550	10565	10580	10595	10609	10624	10639	10654	10668	10683
10500	10649	10663	10678	10693	10707	10722	10737	10751	10766	10780
10600	10747	10762	10776	10791	10805	10820	10834	10849	10863	10878
10700	10846	10860	10875	10889	10904	10918	10932	10947	10961	10975
10800	10945	10959	10973	10988	11002	11016	11030	11044	11058	11073
10900	11043	11058	11072	11086	11100	11114	11128	11142	11156	11170
11000	11142	11156	11170	11184	11198	11212	11226	11240	11254	11268
11100	11241	11255	11269	11283	11296	11310	11324	11338	11352	11365
11200	11340	11353	11367	11381	11395	11408	11422	11436	11449	11463
11300	11438	11452	11466	11479	11493	11507	11520	11534	11547	11561
11400	11537	11551	11564	11578	11591	11605	11618	11632	11645	11659
11500	11636	11649	11663	11676	11690	11703	11716	11730	11743	11756
11600	11735	11748	11762	11775	11788	11801	11815	11828	11841	11854
11700	11834	11847	11860	11873	11886	11900	11913	11926	11939	11952
11800	11933	11946	11959	11972	11985	11998	12011	12024	12037	12050
11900	12031	12044	12057	12070	12083	12096	12109	12122	12135	12148
12000	12130	12143	12156	12169	12182	12195	12208	12220	12233	12246
12100	12229	12242	12255	12268	12280	12293	12306	12319	12331	12344
12200	12328	12341	12354	12366	12379	12392	12404	12417	12429	12442
12300	12427	12440	12452	12465	12478	12490	12503	12515	12528	12540
12400	12526	12539	12551	12564	12576	12589	12601	12613	12626	12638
12500	12625	12638	12650	12662	12675	12687	12699	12712	12724	12736
12600	12724	12737	12749	12761	12773	12786	12798	12810	12822	12834
12700	12823	12835	12848	12860	12872	12884	12896	12908	12921	12933
12800	12922	12934	12947	12959	12971	12983	12995	13007	13019	13031
12900	13021	13033	13045	13057	13069	13081	13093	13105	13117	13129
13000	13120	13132	13144	13156	13168	13180	13192	13204	13216	13227

TABLE IV (Continued)

V_E (FT/SEC) AS A FUNCTION OF ALL-BURNT HEIGHT AND VELOCITY

VEL.	ALL-BURNT HEIGHT. THOUSANDS OF FEET.									
FT/SEC	300	305	310	315	320	325	330	335	340	345
7000	7435	7456	7476	7497	7518	7539	7559	7580	7601	7621
7100	7529	7550	7570	7591	7611	7632	7652	7672	7693	7713
7200	7623	7644	7664	7684	7705	7725	7745	7765	7785	7805
7300	7718	7738	7758	7778	7798	7818	7838	7858	7878	7897
7400	7812	7832	7852	7872	7892	7912	7931	7951	7970	7990
7500	7907	7927	7947	7966	7986	8005	8025	8044	8063	8083
7600	8002	8022	8041	8060	8080	8099	8118	8137	8156	8175
7700	8097	8116	8136	8155	8174	8193	8212	8231	8250	8268
7800	8192	8211	8230	8249	8268	8287	8306	8324	8343	8362
7900	8288	8306	8325	8344	8363	8381	8400	8418	8437	8455
8000	8383	8402	8420	8439	8457	8475	8494	8512	8530	8549
8100	8478	8497	8515	8533	8552	8570	8588	8606	8624	8642
8200	8574	8592	8610	8628	8647	8664	8682	8700	8718	8736
8300	8670	8688	8706	8724	8741	8759	8777	8795	8812	8830
8400	8766	8783	8801	8819	8836	8854	8872	8889	8907	8924
8500	8861	8879	8897	8914	8932	8949	8966	8984	9001	29268
8600	8957	8975	8992	9009	9027	9044	9061	9078	9096	9113
8700	9053	9071	9088	9105	9122	9139	9156	9173	9190	9207
8800	9150	9167	9184	9201	9218	9234	9251	9268	9285	9302
8900	9246	9263	9279	9296	9313	9330	9346	9363	9380	9396
9000	9342	9359	9375	9392	9409	9425	9442	9458	9475	9491
9100	9438	9455	9471	9488	9504	9521	9537	9553	9570	9586
9200	9535	9551	9568	9584	9600	9616	9632	9649	9665	9681
9300	9631	9648	9664	9680	9696	9712	9728	9744	9760	9776
9400	9728	9744	9760	9776	9792	9808	9824	9840	9855	9871
9500	9825	9841	9856	9872	9888	9904	9919	9935	9951	9966
9600	9921	9937	9953	9968	9984	10000	10015	10031	10046	10062
9700	10018	10034	10049	10065	10080	10096	10111	10127	10142	10157
9800	10115	10130	10146	10161	10177	10192	10207	10222	10238	10253
9900	10212	10227	10242	10258	10273	10288	10303	10318	10333	10348
10000	10309	10324	10339	10354	10369	10384	10399	10414	10429	10444
10100	10406	10421	10436	10451	10466	10481	10495	10510	10525	10540
10200	10503	10518	10533	10548	10562	10577	10592	10606	10621	10636
10300	10600	10615	10630	10644	10659	10674	10688	10703	10717	10732
10400	10697	10712	10727	10741	10756	10770	10784	10799	10813	10828
10500	10795	10809	10824	10838	10852	10867	10881	10895	10910	10924
10600	10892	10906	10921	10935	10949	10963	10977	10992	11006	11020
10700	10989	11003	11018	11032	11046	11060	11074	11088	11102	11116
10800	11087	11101	11115	11129	11143	11157	11171	11185	11199	11212
10900	11184	11198	11212	11226	11240	11254	11267	11281	11295	11309
11000	11282	11295	11309	11323	11337	11351	11364	11378	11392	11405
11100	11379	11393	11407	11420	11434	11447	11461	11475	11488	11502
11200	11477	11490	11504	11517	11531	11544	11558	11571	11585	11598
11300	11574	11588	11601	11615	11628	11641	11655	11668	11682	11695
11400	11672	11685	11699	11712	11725	11739	11752	11765	11778	11791
11500	11770	11783	11796	11809	11823	11836	11849	11862	11875	11888
11600	11867	11881	11894	11907	11920	11933	11946	11959	11972	11985
11700	11965	11978	11991	12004	12017	12030	12043	12056	12069	12082
11800	12063	12076	12089	12102	12115	12127	12140	12153	12166	12179
11900	12161	12174	12186	12199	12212	12225	12237	12250	12263	12276
12000	12259	12271	12284	12297	12309	12322	12335	12347	12360	12373
12100	12357	12369	12382	12394	12407	12420	12432	12445	12457	12470
12200	12455	12467	12480	12492	12505	12517	12529	12542	12554	12567
12300	12553	12565	12577	12590	12602	12614	12627	12639	12651	12664
12400	12651	12663	12675	12687	12700	12712	12724	12736	12749	12761
12500	12749	12761	12773	12785	12797	12810	12822	12834	12846	12858
12600	12847	12859	12871	12883	12895	12907	12919	12931	12943	12955
12700	12945	12957	12969	12981	12993	13005	13017	13029	13041	13053
12800	13043	13055	13067	13079	13091	13102	13114	13126	13138	13150
12900	13141	13153	13165	13177	13188	13200	13212	13224	13236	13247
13000	13239	13251	13263	13274	13286	13298	13310	13321	13333	13345

TABLE IV (Continued)

V_E (FT/SEC) AS A FUNCTION OF ALL-BURNT HEIGHT AND VELOCITY

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 350	355	360	365	370	375	380	385	390	395
7000	7641	7662	7682	7702	7722	7742	7762	7782	7802	7822
7100	7733	7753	7773	7793	7813	7833	7853	7872	7892	7911
7200	7825	7845	7865	7884	7904	7923	7943	7962	7982	8001
7300	7917	7937	7956	7976	7995	8014	8034	8053	8072	8091
7400	8009	8029	8048	8067	8086	8106	8125	8144	8163	8182
7500	8102	8121	8140	8159	8178	8197	8216	8235	8253	8272
7600	8194	8213	8232	8251	8270	8289	8307	8326	8344	8363
7700	8287	8306	8325	8343	8362	8380	8399	8417	8436	8454
7800	8380	8399	8417	8436	8454	8472	8491	8509	8527	8545
7900	8473	8492	8510	8528	8546	8565	8583	8601	8619	8637
8000	8567	8585	8603	8621	8639	8657	8675	8693	8710	8728
8100	8660	8678	8696	8714	8732	8749	8767	8785	8802	8820
8200	8754	8772	8789	8807	8824	8842	8860	8877	8894	8912
8300	8848	8865	8883	8900	8917	8935	8952	8969	8987	9004
8400	8941	8959	8976	8993	9011	9028	9045	9062	9079	9096
8500	9035	9053	9070	9087	9104	9121	9138	9155	9172	9189
8600	9130	9147	9164	9180	9197	9214	9231	9248	9264	9281
8700	9224	9241	9258	9274	9291	9308	9324	9341	9357	9374
8800	9318	9335	9352	9368	9385	9401	9418	9434	9450	9467
8900	9413	9429	9446	9462	9478	9495	9511	9527	9544	9560
9000	9507	9524	9540	9556	9572	9589	9605	9621	9637	9653
9100	9602	9618	9634	9650	9666	9683	9699	9714	9730	9746
9200	9697	9713	9729	9745	9761	9777	9792	9808	9824	9840
9300	9792	9808	9824	9839	9855	9871	9886	9902	9918	9933
9400	9887	9903	9918	9934	9949	9965	9981	9996	10012	10027
9500	9982	9998	10013	10029	10044	10059	10075	10090	10105	10121
9600	10077	10093	10108	10123	10139	10154	10169	10184	10200	10215
9700	10172	10188	10203	10218	10233	10248	10264	10279	10294	10309
9800	10268	10283	10298	10313	10328	10343	10358	10373	10388	10403
9900	10363	10378	10393	10408	10423	10438	10453	10468	10482	10497
10000	10459	10474	10489	10503	10518	10533	10548	10562	10577	10592
10100	10555	10569	10584	10599	10613	10628	10642	10657	10671	10686
10200	10650	10665	10679	10694	10708	10723	10737	10752	10766	10781
10300	10746	10761	10775	10789	10804	10818	10832	10847	10861	10875
10400	10842	10856	10871	10885	10899	10913	10928	10942	10956	10970
10500	10938	10952	10966	10980	10995	11009	11023	11037	11051	11065
10600	11034	11048	11062	11076	11090	11104	11118	11132	11146	11160
10700	11130	11144	11158	11172	11186	11200	11213	11227	11241	11255
10800	11226	11240	11254	11268	11281	11295	11309	11323	11336	11350
10900	11323	11336	11350	11364	11377	11391	11404	11418	11432	11445
11000	11419	11432	11446	11460	11473	11487	11500	11514	11527	11540
11100	11515	11529	11542	11556	11569	11582	11596	11609	11622	11636
11200	11612	11625	11638	11652	11665	11678	11692	11705	11718	11731
11300	11708	11721	11735	11748	11761	11774	11787	11800	11814	11827
11400	11805	11818	11831	11844	11857	11870	11883	11896	11909	11922
11500	11901	11914	11927	11940	11953	11966	11979	11992	12005	12018
11600	11998	12011	12024	12037	12050	12062	12075	12088	12101	12114
11700	12095	12107	12120	12133	12146	12159	12171	12184	12197	12209
11800	12191	12204	12217	12230	12242	12255	12268	12280	12293	12305
11900	12288	12301	12313	12326	12339	12351	12364	12376	12389	12401
12000	12385	12398	12410	12423	12435	12448	12460	12472	12485	12497
12100	12482	12494	12507	12519	12532	12544	12556	12569	12581	12593
12200	12579	12591	12604	12616	12628	12640	12653	12665	12677	12689
12300	12676	12688	12700	12713	12725	12737	12749	12761	12773	12786
12400	12773	12785	12797	12809	12822	12834	12846	12858	12870	12882
12500	12870	12882	12894	12906	12918	12930	12942	12954	12966	12978
12600	12967	12979	12991	13003	13015	13027	13039	13051	13063	13074
12700	13064	13076	13088	13100	13112	13124	13136	13147	13159	13171
12800	13162	13173	13185	13197	13209	13221	13232	13244	13256	13267
12900	13259	13271	13282	13294	13306	13317	13329	13341	13352	13364
13000	13356	13368	13380	13391	13403	13414	13426	13437	13449	13460

TABLE IV (Continued)

V_E (FT/SEC) AS A FUNCTION OF ALL-BURNT HEIGHT AND VELOCITY

VEL. FT/SEC	ALL-BURNT HEIGHT. THOUSANDS OF FEET. 400	405	410	415	420	425	430	435	440	445
7000	7842	7861	7881	7900	7920	7939	7959	7978	7998	8017
7100	7931	7950	7970	7989	8009	8028	8047	8066	8085	8104
7200	8021	8040	8059	8078	8097	8116	8135	8154	8173	8192
7300	8111	8129	8148	8167	8186	8205	8224	8243	8261	8280
7400	8201	8219	8238	8257	8276	8294	8313	8331	8350	8368
7500	8291	8310	8328	8347	8365	8384	8402	8420	8439	8457
7600	8382	8400	8418	8437	8455	8473	8491	8509	8528	8546
7700	8472	8490	8509	8527	854	8563	8581	8599	8617	8635
7800	8563	8581	8599	8617	8635	8653	8671	8689	8706	8724
7900	8655	8672	8690	8708	8726	8743	8761	8778	8796	8813
8000	8746	8763	8781	8799	8816	8834	8851	8869	8886	8903
8100	8837	8855	8872	8890	8907	8924	8942	8959	8976	8993
8200	8929	8946	8964	8981	8998	9015	9032	9049	9066	9083
8300	9021	9038	9055	9072	9089	9106	9123	9140	9157	9174
8400	9113	9130	9147	9164	9181	9198	9214	9231	9248	9264
8500	9205	9222	9239	9256	9272	9289	9306	9322	9339	9355
8600	9298	9314	9331	9348	9364	9381	9397	9413	9430	9446
8700	9390	9407	9423	9440	9456	9472	9489	9505	9521	9537
8800	9483	9499	9516	9532	9548	9564	9580	9596	9613	9628
8900	9576	9592	9608	9624	9640	9656	9672	9688	9704	9720
9000	9669	9685	9701	9717	9733	9749	9764	9780	9796	9812
9100	9762	9778	9794	9810	9825	9841	9857	9872	9888	9903
9200	9855	9871	9887	9902	9918	9933	9949	9964	9980	9995
9300	9949	9964	9980	9995	10011	10026	10042	10057	10072	10087
9400	10042	10058	10073	10088	10104	10119	10134	10149	10165	10180
9500	10136	10151	10167	10182	10197	10212	10227	10242	10257	10272
9600	10230	10245	10260	10275	10290	10305	10320	10335	10350	10365
9700	10324	10339	10354	10369	10383	10398	10413	10428	10443	10457
9800	10418	10433	10447	10462	10477	10492	10506	10521	10536	10550
9900	10512	10527	10541	10556	10571	10585	10600	10614	10629	10643
10000	10606	10621	10635	10650	10664	10679	10693	10708	10722	10736
10100	10700	10715	10729	10744	10758	10772	10787	10801	10815	10829
10200	10795	10809	10823	10838	10852	10866	10880	10895	10909	10923
10300	10889	10904	10918	10932	10946	10960	10974	10988	11002	11016
10400	10984	10998	11012	11026	11040	11054	11068	11082	11096	11110
10500	11079	11093	11107	11121	11134	11148	11162	11176	11190	11203
10600	11174	11187	11201	11215	11229	11243	11256	11270	11284	11297
10700	11269	11282	11296	11310	11323	11337	11350	11364	11378	11391
10800	11364	11377	11391	11404	11418	11431	11445	11458	11472	11485
10900	11459	11472	11486	11499	11512	11526	11539	11553	11566	11579
11000	11554	11567	11581	11594	11607	11620	11634	11647	11660	11673
11100	11649	11662	11676	11689	11702	11715	11728	11741	11755	11768
11200	11744	11758	11771	11784	11797	11810	11823	11836	11849	11862
11300	11840	11853	11866	11879	11892	11905	11918	11931	11944	11957
11400	11935	11948	11961	11974	11987	12000	12013	12025	12038	12051
11500	12031	12044	12056	12069	12082	12095	12108	12120	12133	12146
11600	12126	12139	12152	12165	12177	12190	12203	12215	12228	12240
11700	12222	12235	12247	12260	12273	12285	12298	12310	12323	12335
11800	12318	12330	12343	12355	12368	12380	12393	12405	12418	12430
11900	12414	12426	12439	12451	12463	12476	12488	12500	12513	12525
12000	12510	12522	12534	12547	12559	12571	12583	12596	12608	12620
12100	12606	12618	12630	12642	12655	12667	12679	12691	12703	12715
12200	12702	12714	12726	12738	12750	12762	12774	12786	12798	12810
12300	12798	12810	12822	12834	12846	12858	12870	12882	12894	12906
12400	12894	12906	12918	12930	12942	12954	12965	12977	12989	13001
12500	12990	13002	13014	13026	13038	13049	13061	13073	13085	13097
12600	13086	13098	13110	13122	13133	13145	13157	13169	13180	13192
12700	13183	13194	13206	13218	13229	13241	13253	13264	13276	13288
12800	13279	13291	13302	13314	13325	13337	13349	13360	13372	13383
12900	13375	13387	13398	13410	13422	13433	13444	13456	13467	13479
13000	13472	13483	13495	13506	13518	13529	13540	13552	13563	13575

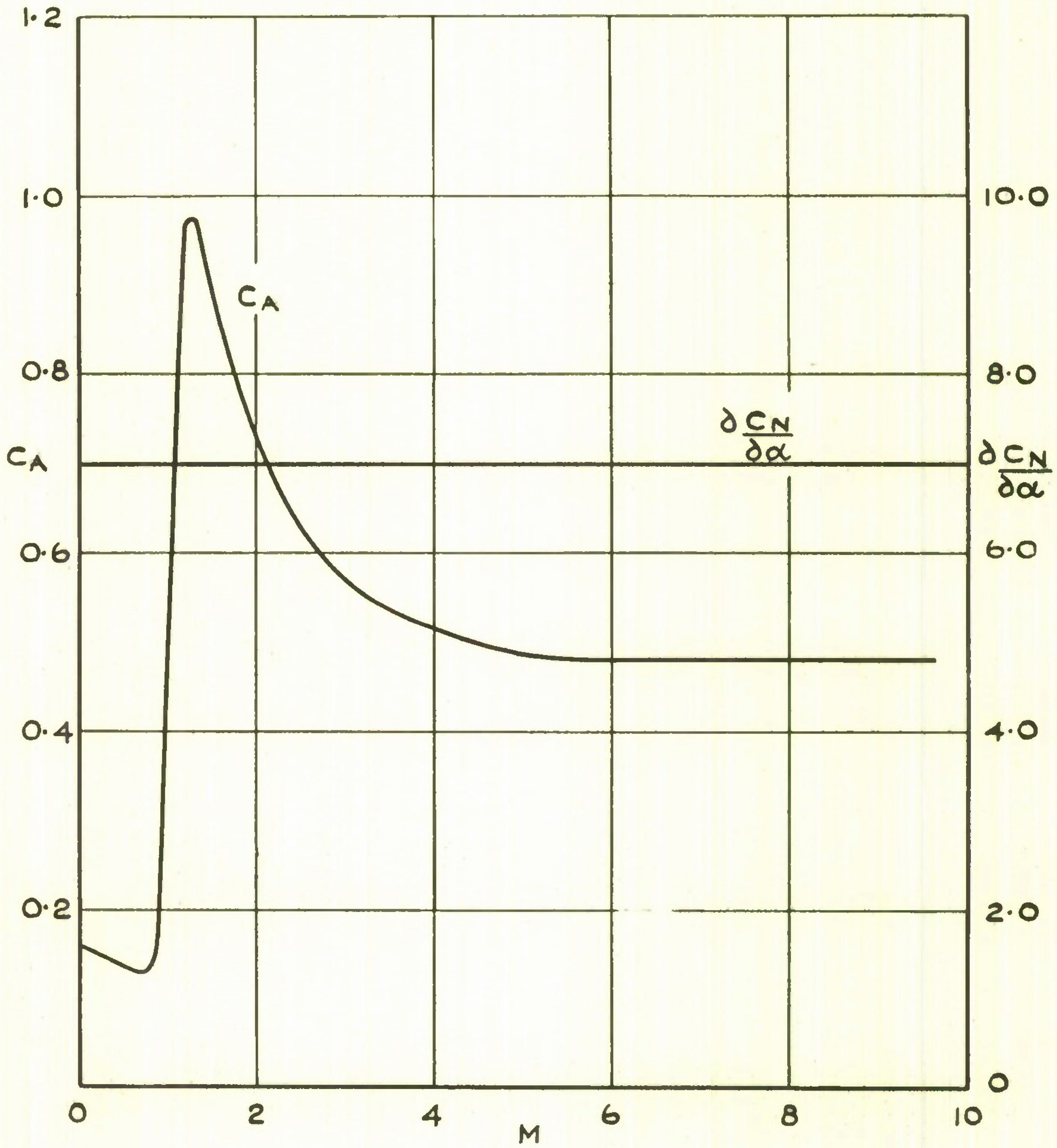
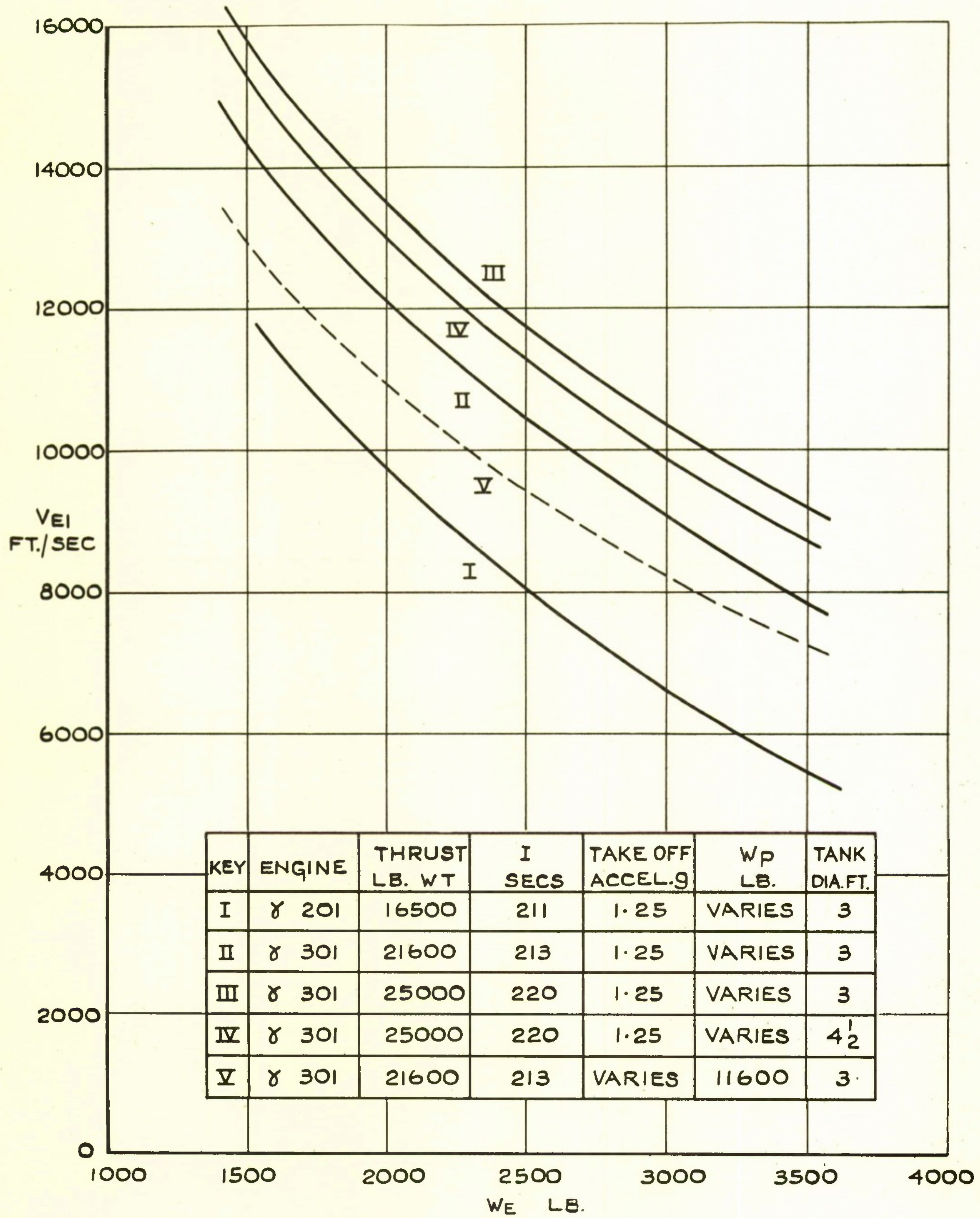


FIG. 1. VARIATION OF C_N AND $\frac{\partial C_N}{\partial \alpha}$ FOR THE FIRST STAGE OF BLACK KNIGHT.



KEY	ENGINE	THRUST LB. WT	I SECS	TAKE OFF ACCEL. g	W_P LB.	TANK DIA. FT.
I	γ 201	16500	211	1·25	VARIES	3
II	γ 301	21600	213	1·25	VARIES	3
III	γ 301	25000	220	1·25	VARIES	3
IV	γ 301	25000	220	1·25	VARIES	$4\frac{1}{2}$
V	γ 301	21600	213	VARIES	11600	3

FIG. 2. VARIATION OF FIRST STAGE RE-ENTRY VELOCITY V_{EI} WITH EMPTY WEIGHT W_E FOR USEFUL FORMS OF BLACK KNIGHT.

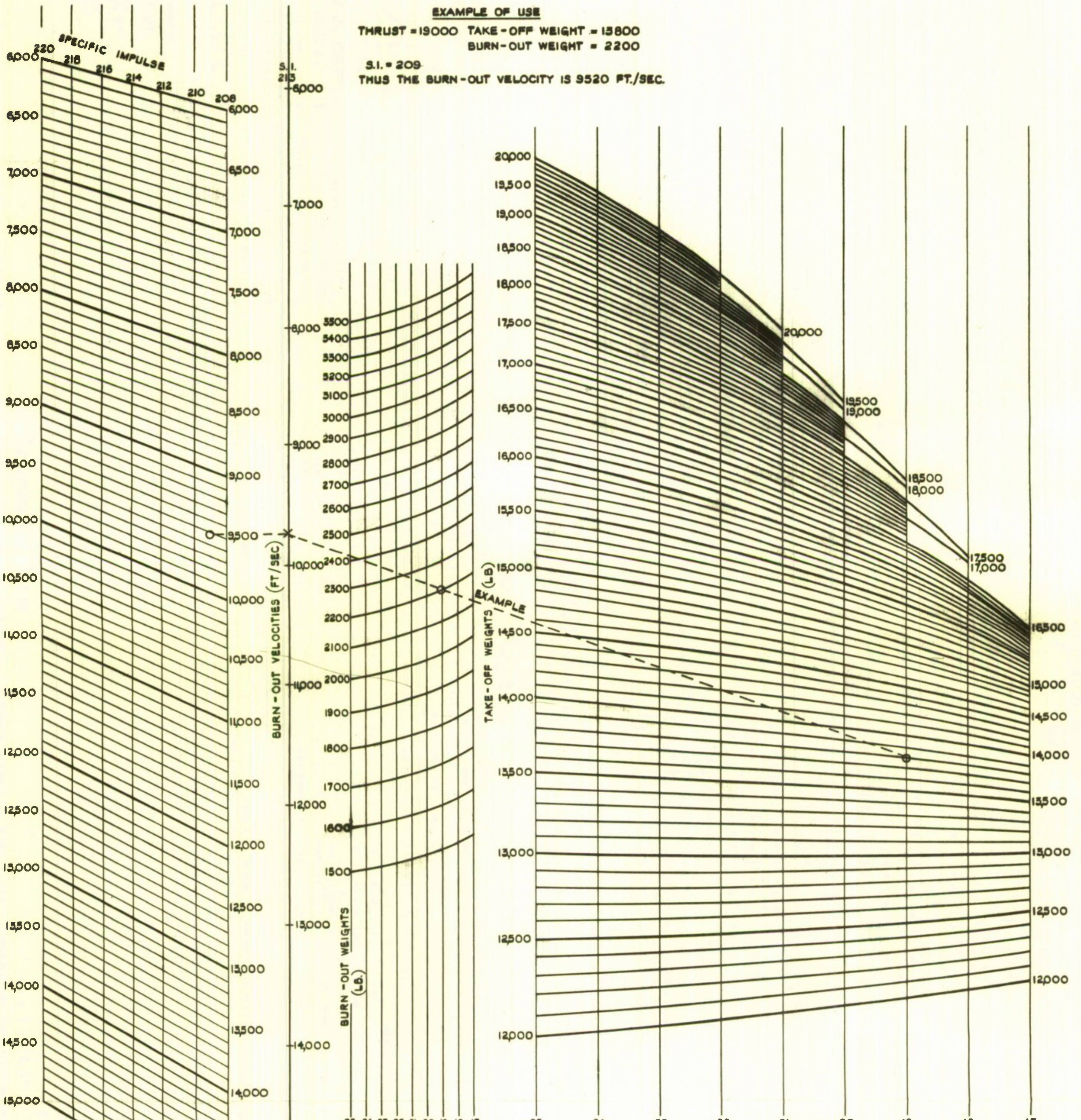


FIG. 3. NOMOGRAM FOR BURN-OUT VELOCITY, THRUST, SPECIFIC IMPULSE, TAKE-OFF WEIGHT AND BURN-OUT WEIGHT.
FOR A 36 INS. DIAM. "BLACK KNIGHT" VEHICLE WITH A GAMMA 301 ENGINE

RESTRICTED

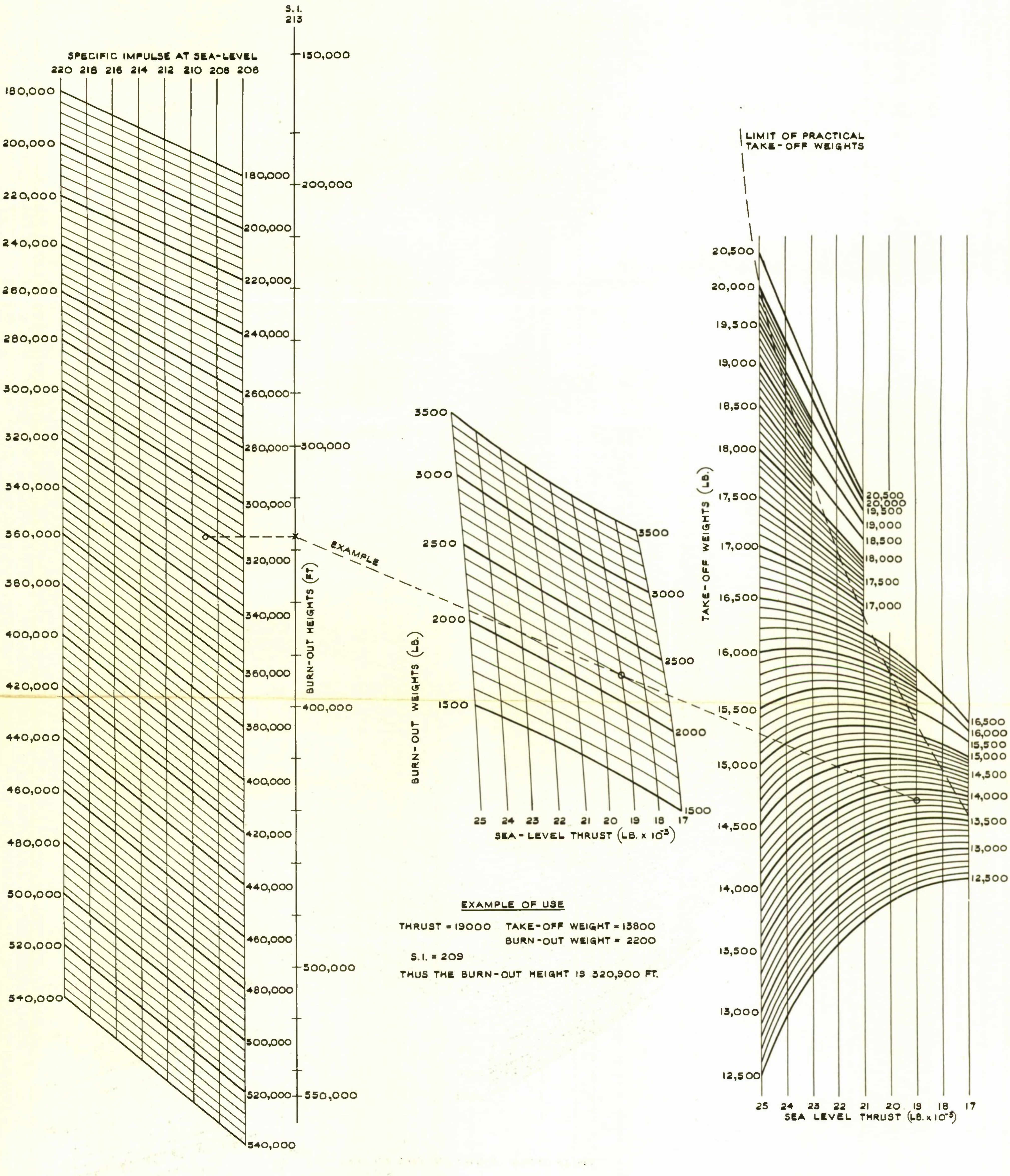


FIG.4. NOMOGRAM FOR BURN-OUT HEIGHT, THRUST, SPECIFIC IMPULSE, TAKE-OFF WEIGHT AND BURN-OUT WEIGHT.
FOR A 36 INS. DIAM. "BLACK KNIGHT" VEHICLE WITH A GAMMA 301 ENGINE

RESTRICTED

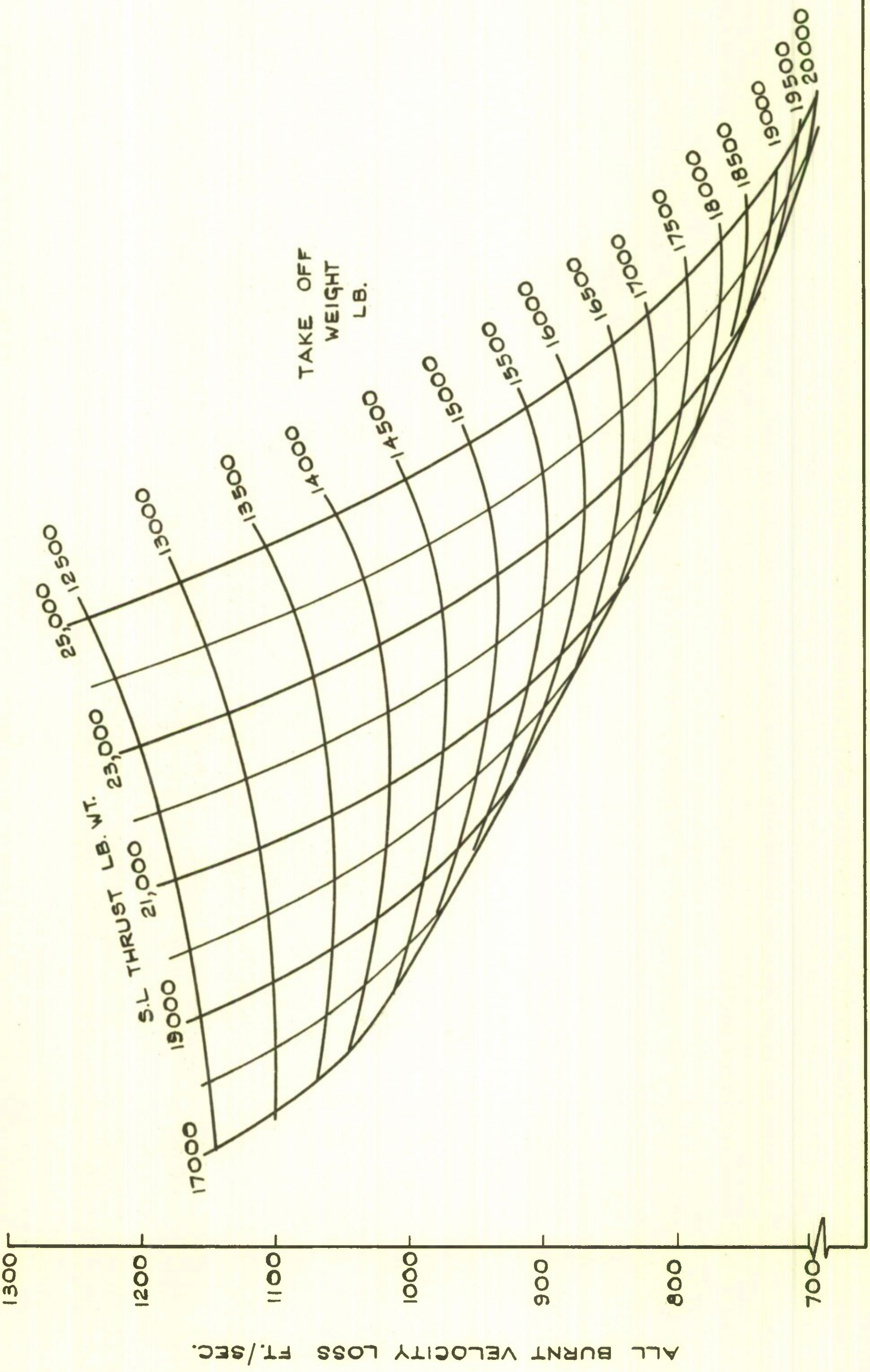


FIG. 5. LOSS OF VELOCITY AT ALL BURNT OF THE FIRST STAGE DUE TO THE PRESENCE OF THE ATMOSPHERE AS A FUNCTION OF TAKE-OFF. THRUST AND WEIGHT (3 FT. VEHICLE γ 301 ENGINE) S.I. = 213 SECS.

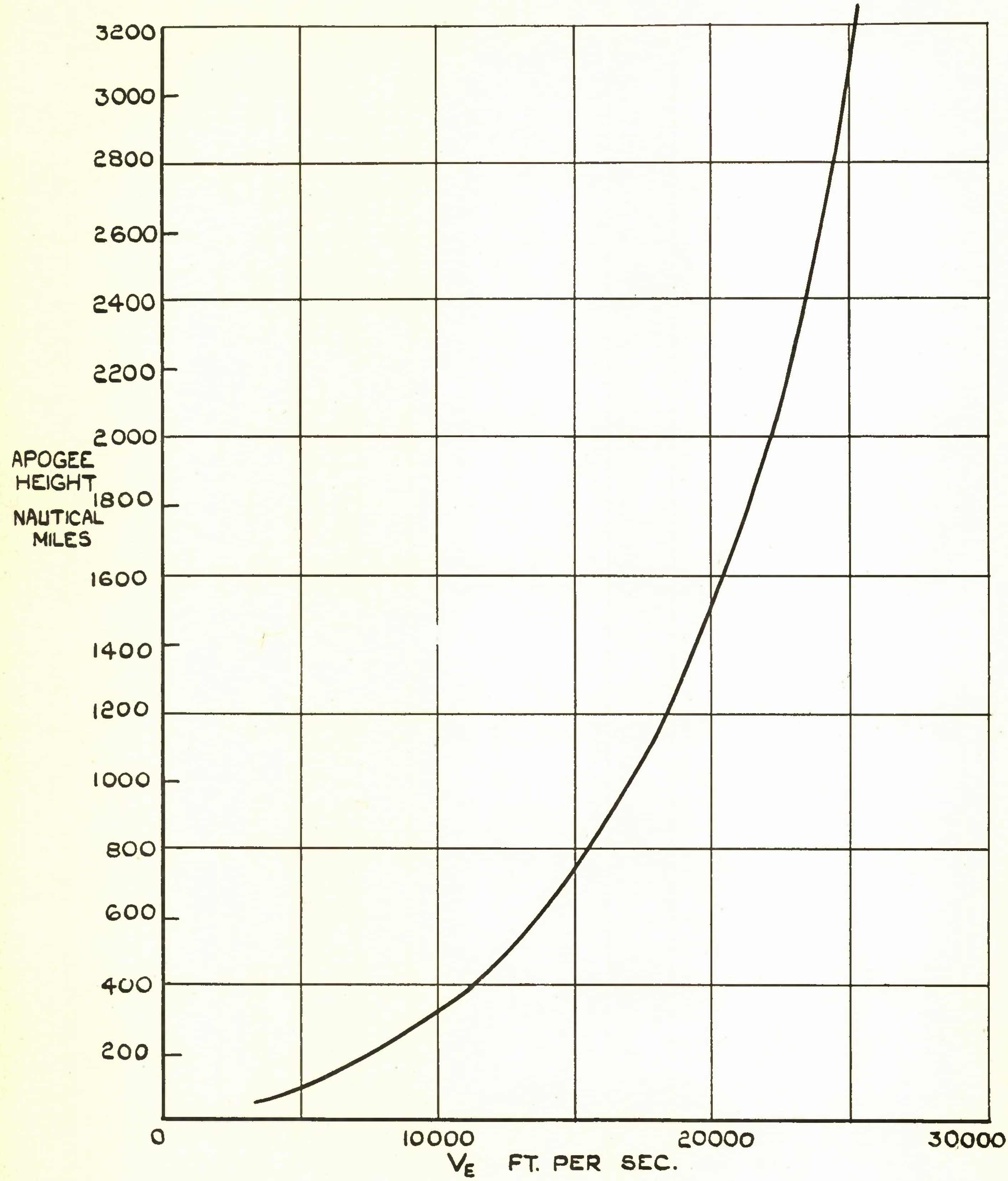


FIG. 6. APOGEE HEIGHT AS A FUNCTION OF RE-ENTRY VELOCITY V_E.

FIG. 7.

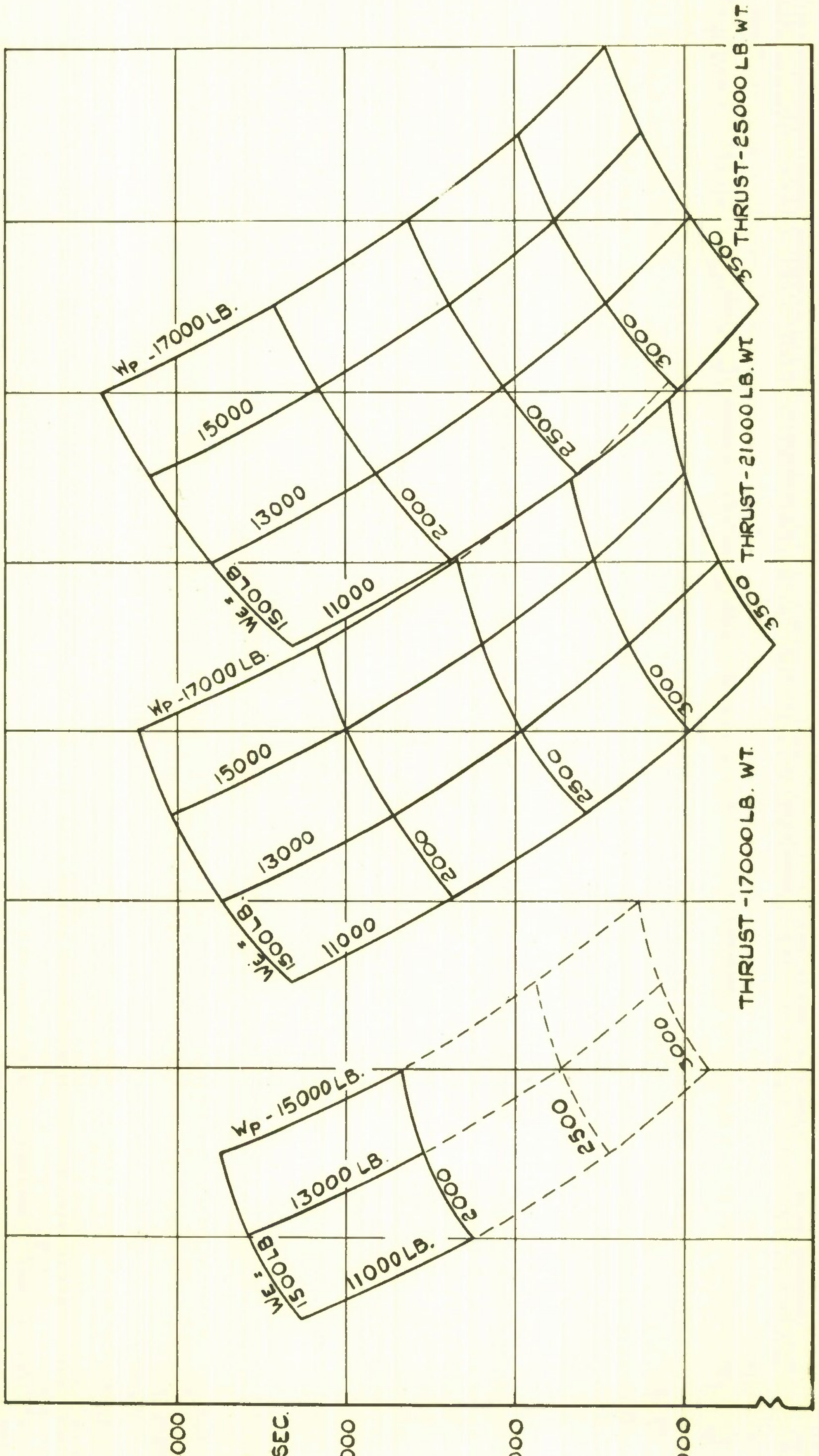


FIG. 7. VARIATION OF FIRST STAGE RE-ENTRY VELOCITY V_{E1} WITH EMPTY WEIGHT, PROPELLANT WEIGHT & THRUST FOR A SPECIFIC IMPULSE OF 213 ON A 36" VEHICLE.

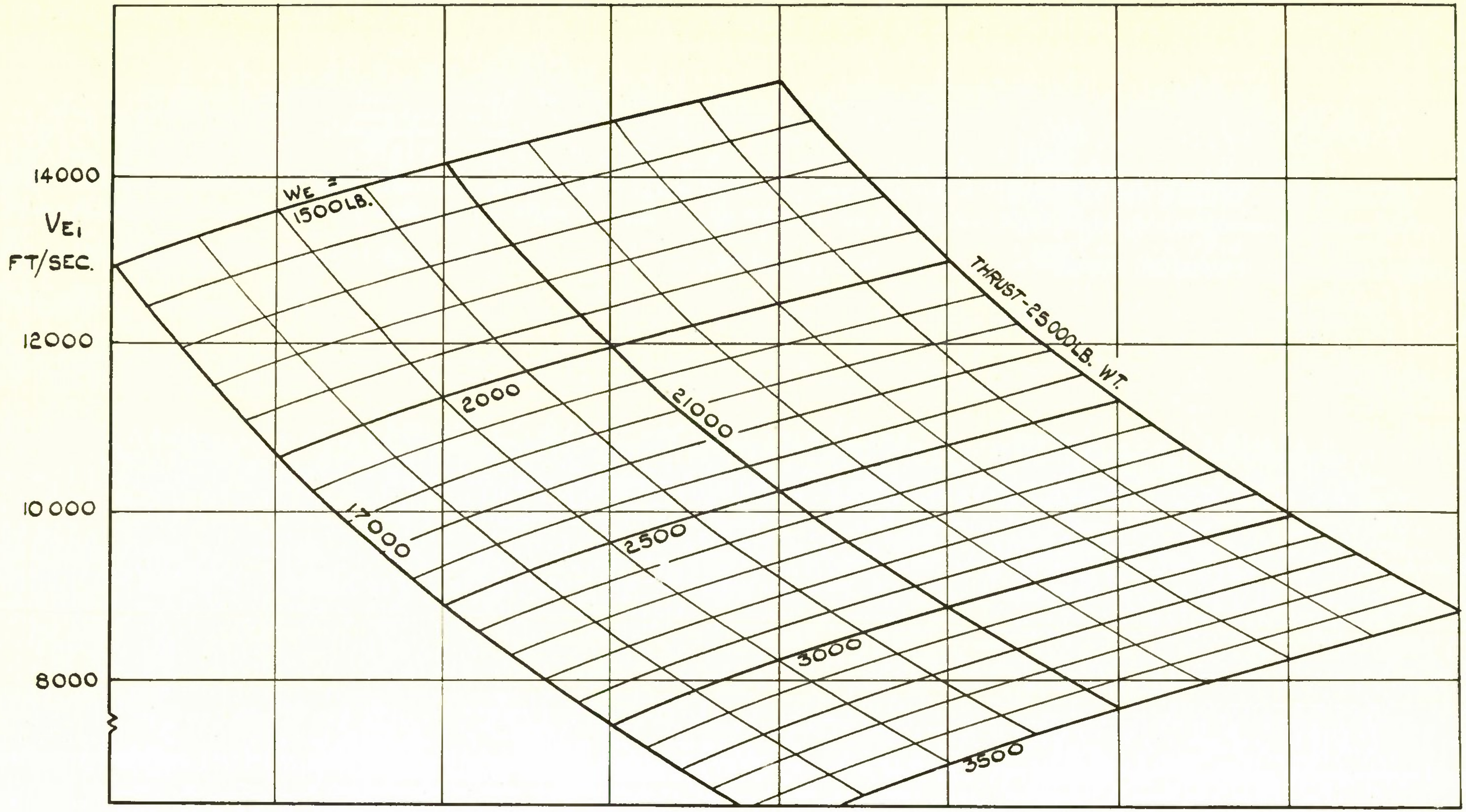


FIG. 8. VARIATION OF FIRST STAGE RE-ENTRY VELOCITY V_{E1} WITH EMPTY WEIGHT & THRUST FOR A SPECIFIC IMPULSE OF 213 & TAKE OFF THRUST WEIGHT RATIO OF 1·25.

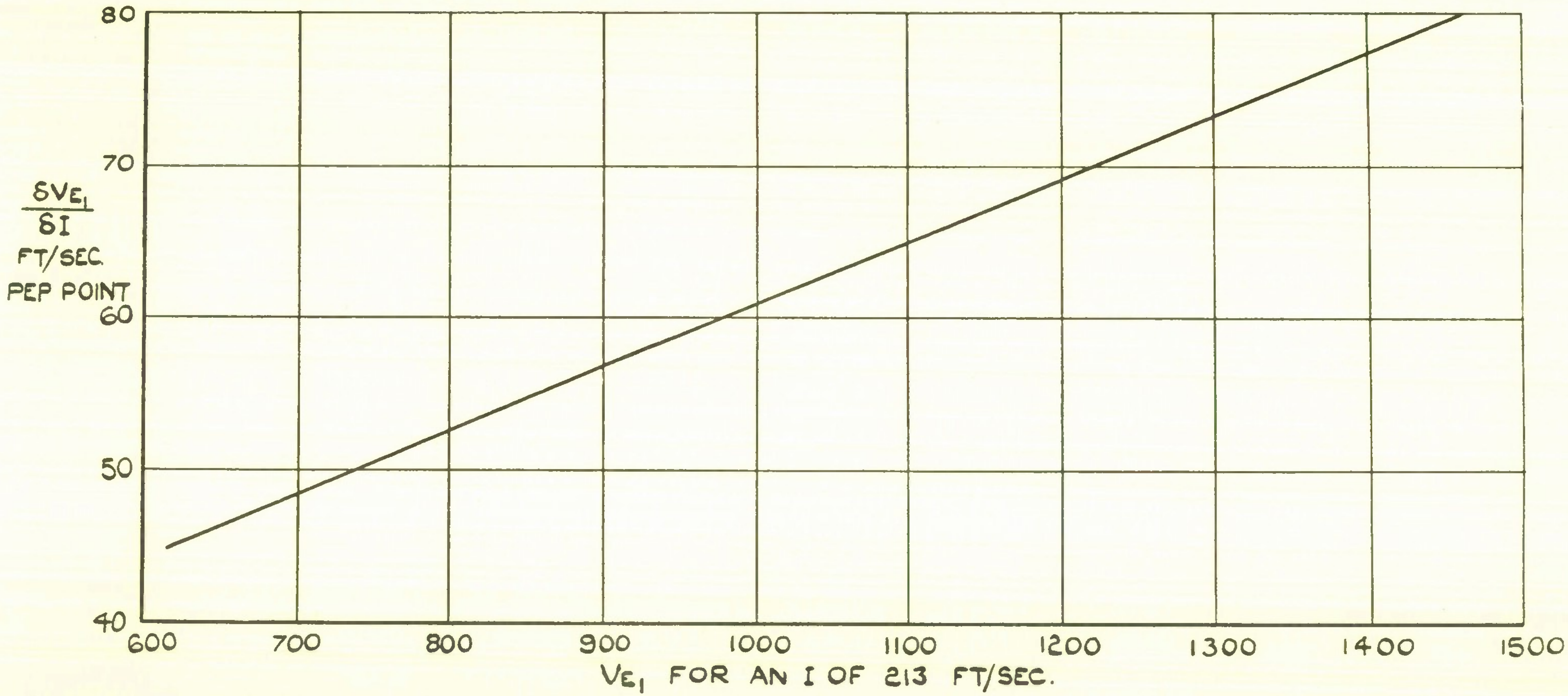


FIG. 9. RATE OF CHANGE OF FIRST STAGE RE-ENTRY VELOCITY V_{E_1} WITH SPECIFIC IMPULSE I AS A FUNCTION OF THE V_{E_1} ACHIEVED WITH AN I OF 213

FIG. 10.

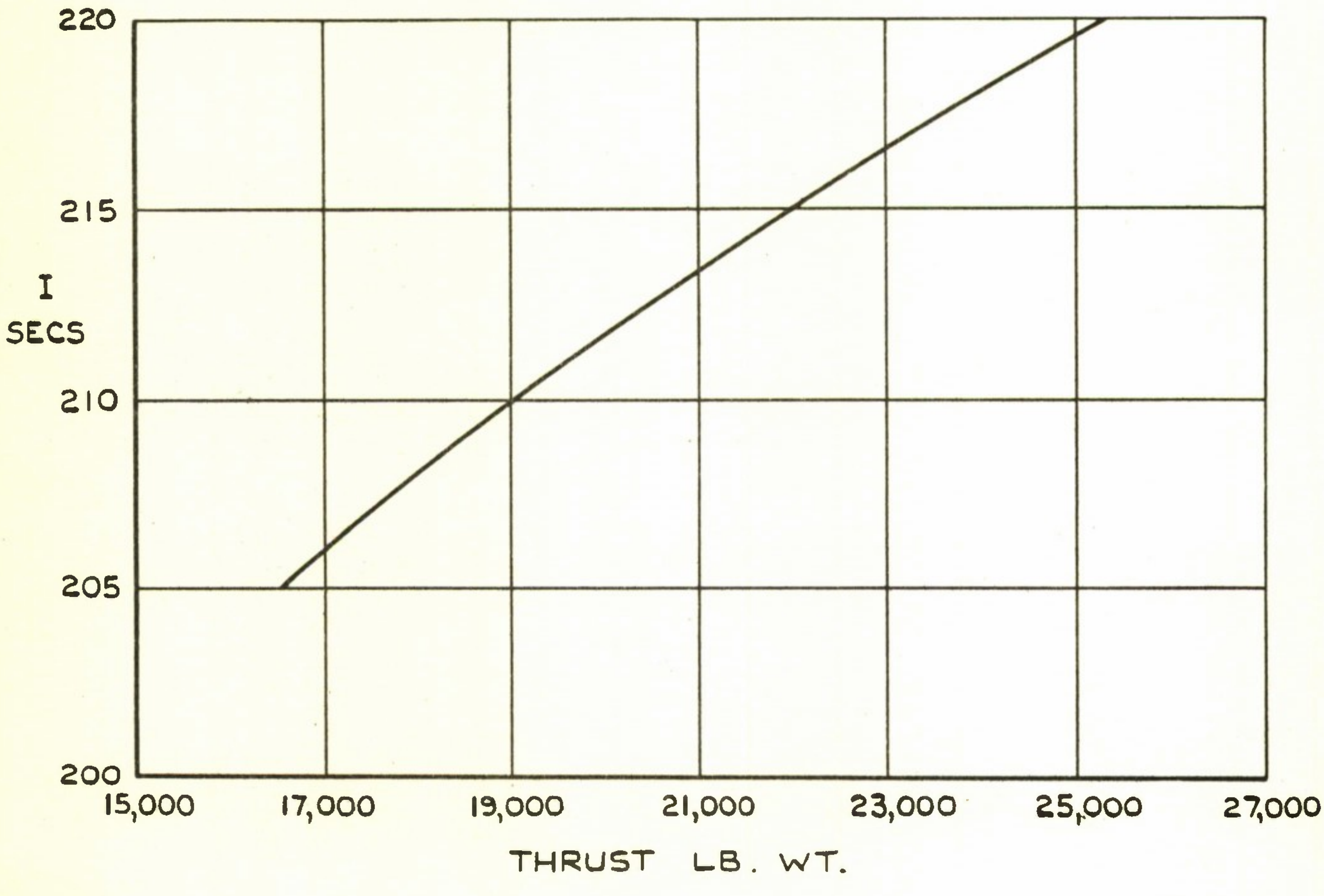
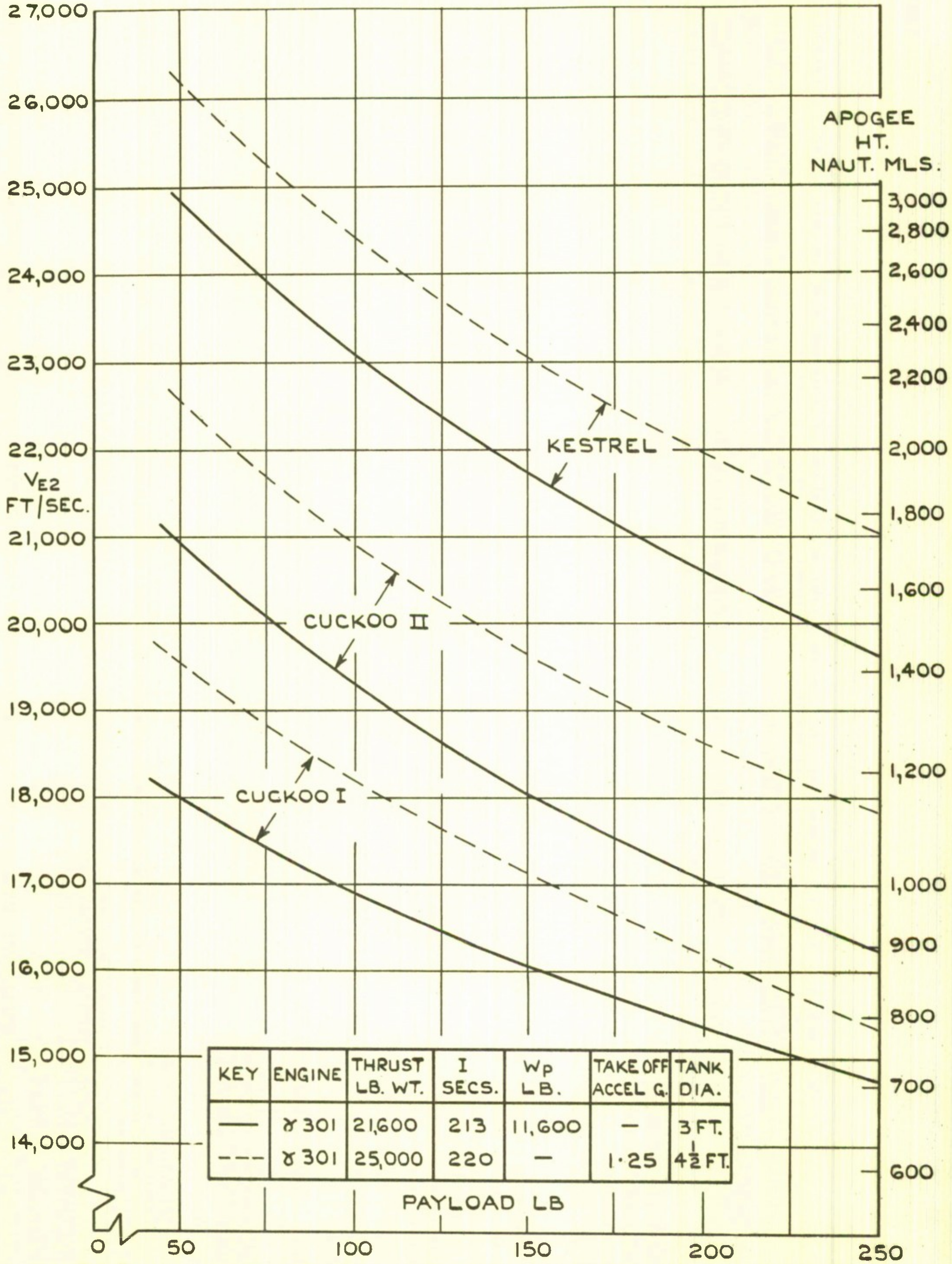


FIG. 10. VARIATION OF SPECIFIC IMPULSE I WITH THRUST T OF THE γ301 ENGINE AT SEA LEVEL



KEY	ENGINE	THRUST LB. WT.	I SECS.	Wp LB.	TAKE OFF ACCEL G.	TANK DIA.
——	γ 301	21,600	213	11,600	—	3 FT.
- - -	γ 301	25,000	220	—	1·25	4½ FT.

FIG. 11. VARIATION OF RE-ENTRY VELOCITY V_{E2} WITH PAYLOAD FOR A RANGE OF TWO STAGE CONFIGURATIONS. BOTH STAGES FIRING UPWARDS.

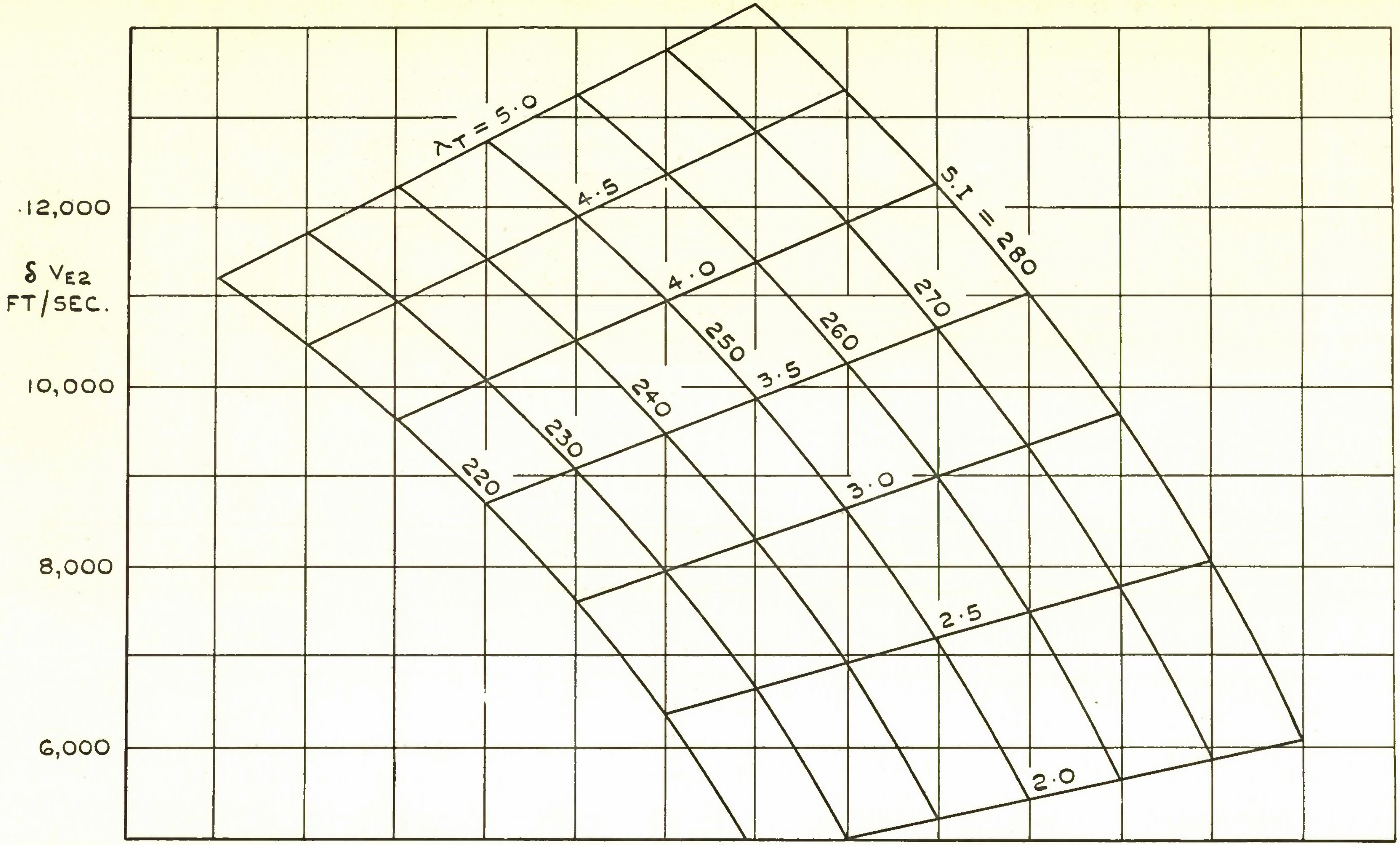


FIG. 12. RE-ENTRY VELOCITY INCREMENT OF SECOND STAGE AS A FUNCTION OF WEIGHT RATIO λT & SPECIFIC IMPULSE OF THE SECOND STAGE WHEN FIRED DOWNWARDS AT 350,000 FT.

FIG. 13.(a)

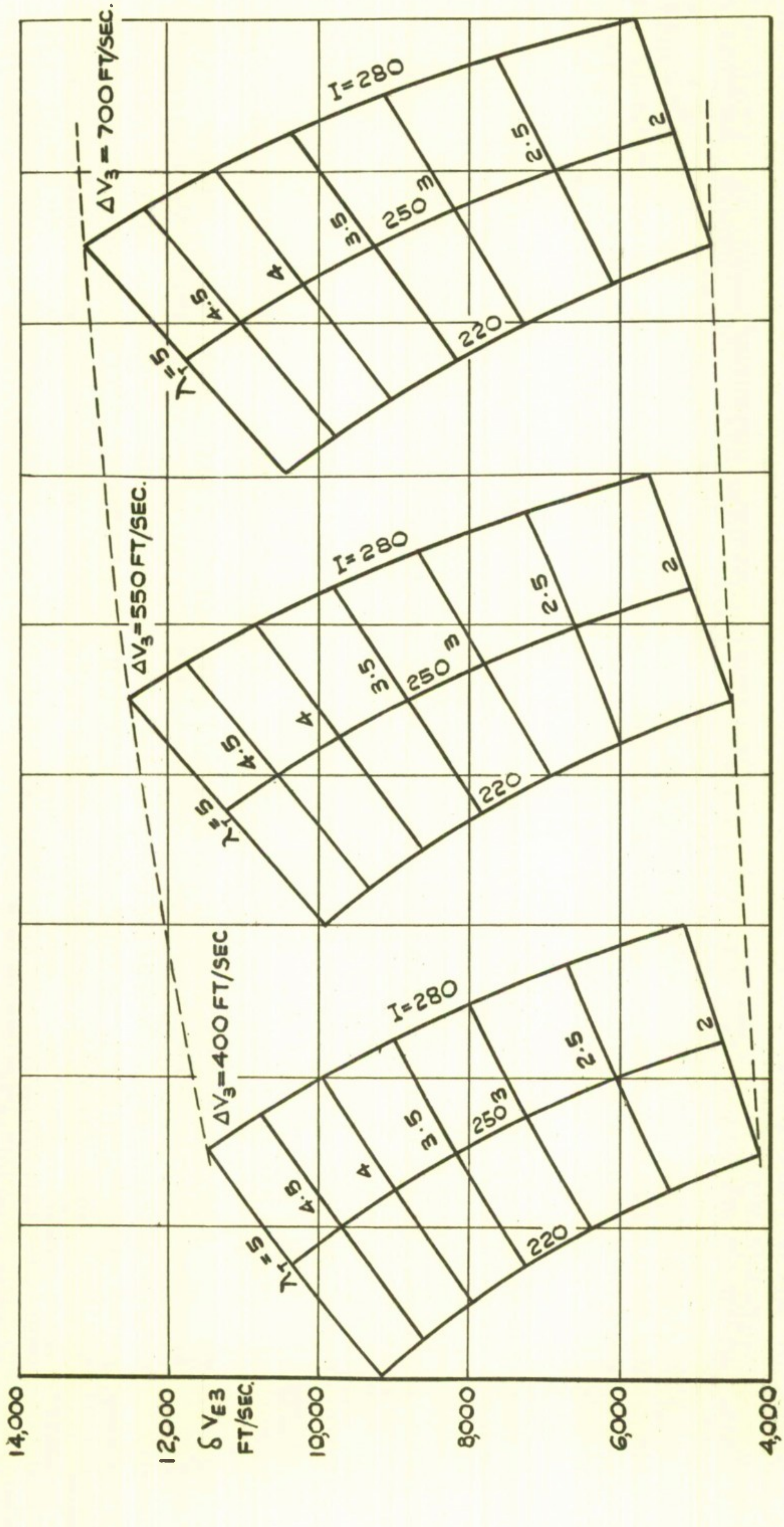


FIG. 13. (a) RE-ENTRY VELOCITY INCREMENT OF SECOND PLUS THIRD STAGE AS A FUNCTION OF SECOND STAGE WEIGHT RATIO λ_T & SPECIFIC IMPULSE I & THIRD STAGE VELOCITY INCREMENT FOR A FIRST STAGE V_{E_1} OF 9500 FT / SECOND.

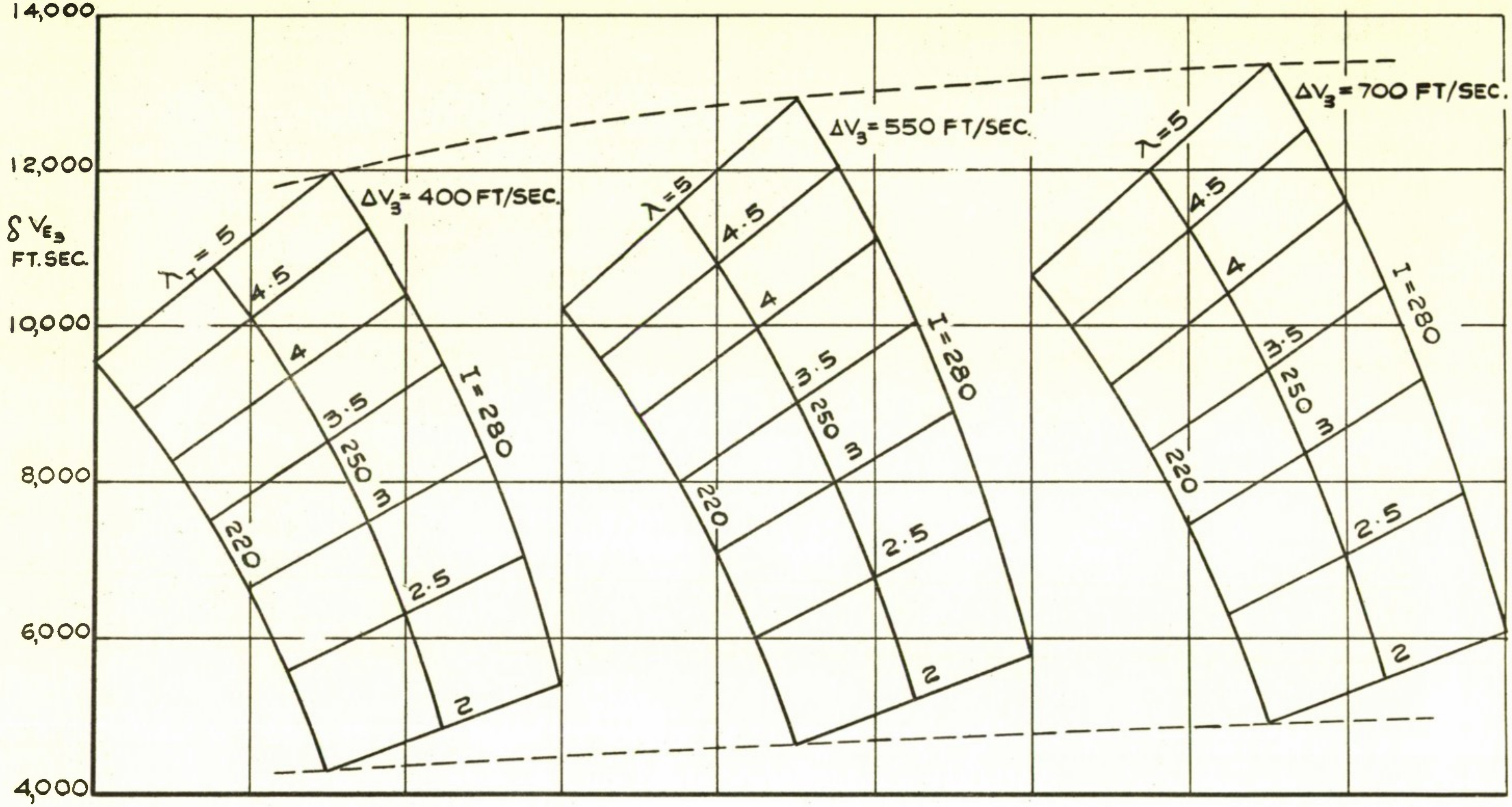
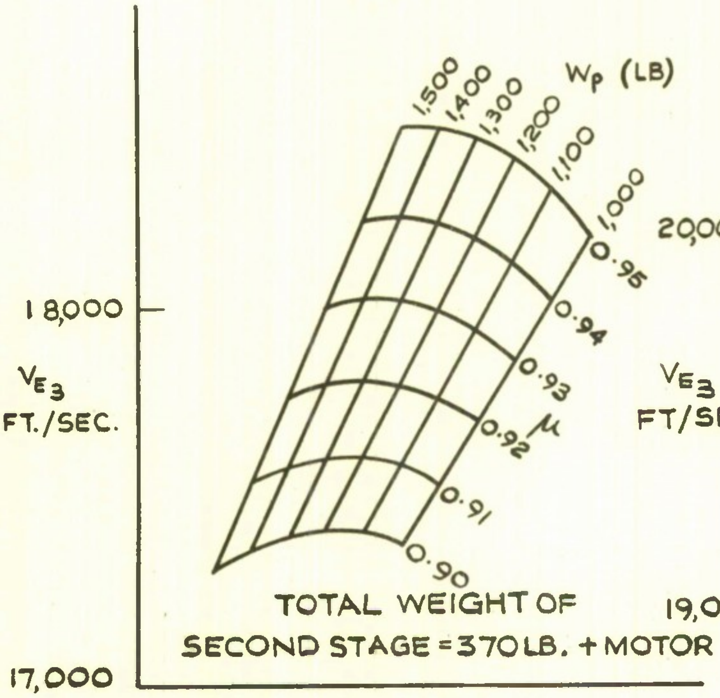
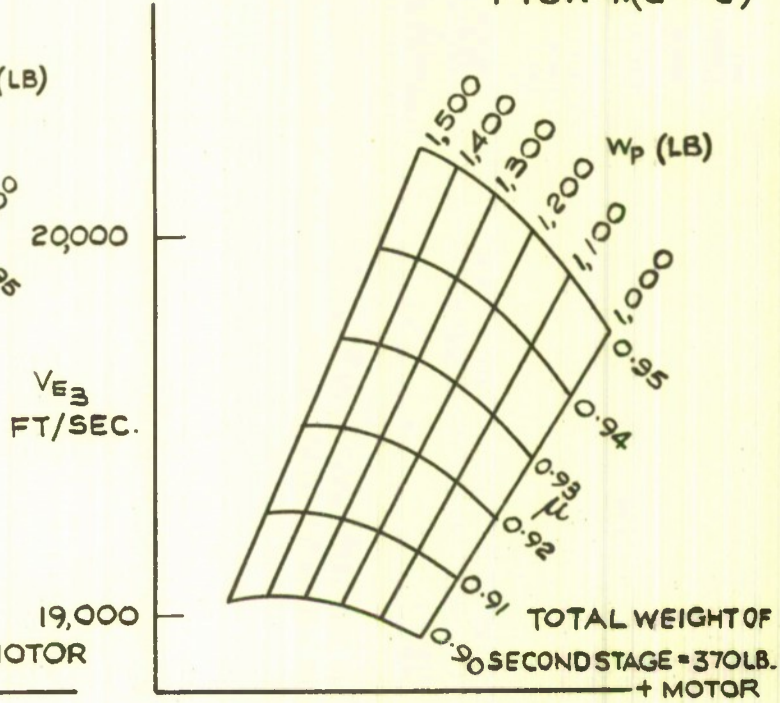


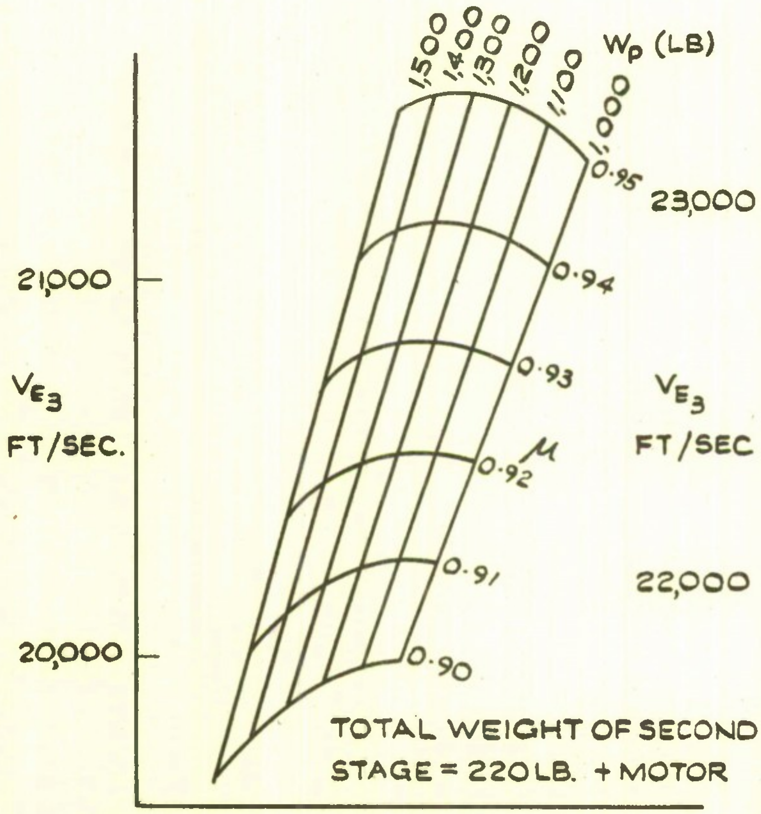
FIG. 13. (b) RE-ENTRY VELOCITY INCREMENT OF SECOND PLUS THIRD STAGE AS A FUNCTION OF SECOND STAGE WEIGHT RATIO λ_T & SPECIFIC IMPULSE I & THIRD STAGE VELOCITY INCREMENT FOR A FIRST STAGE V_{E_1} OF 10,500 FT/SEC.



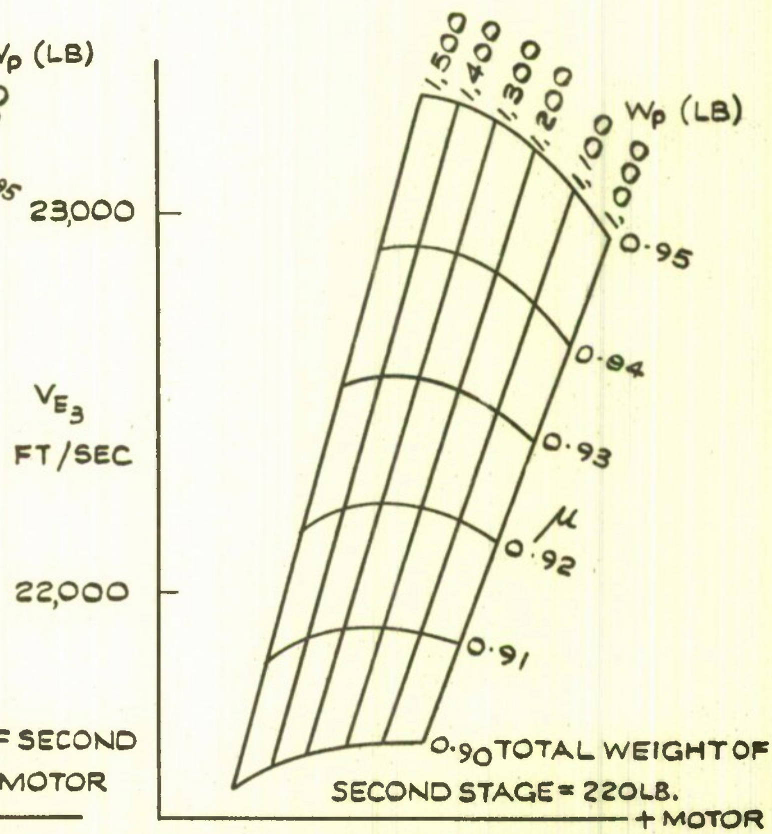
(a) PAYLOAD WEIGHT 250 LB.
FIRST STAGE THRUST = 21,600 LB. WT.
I = 213. W_p = 11,600 LB. W_E = 1380 LB.



(b) PAYLOAD WEIGHT 250 LB.
FIRST STAGE T = 25,000 LB. WT.
4½' DIA. TANKS
I = 220, W_p = 15,600 LB. W_E = 1480 LB.

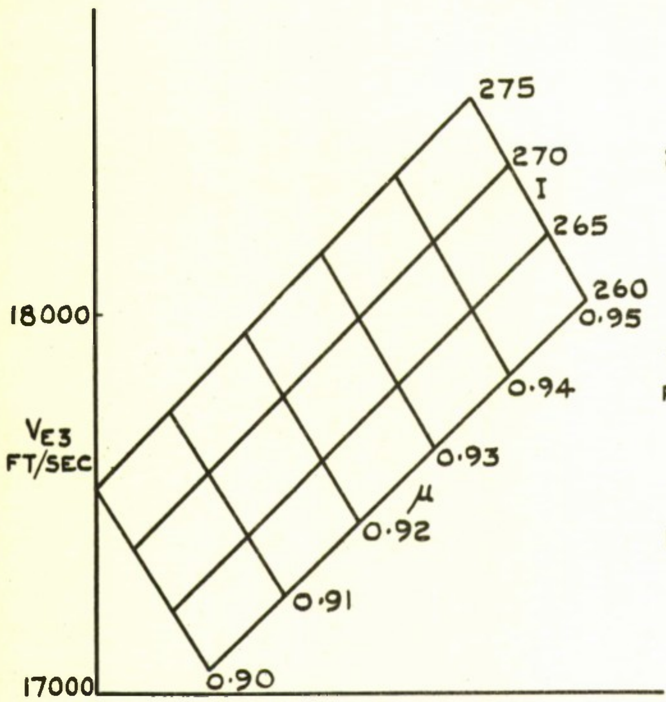


(c) PAYLOAD WEIGHT 110 LB
FIRST STAGE T = 21600 LB. WT.
I = 213, W_p = 11,600 LB. W_E = 1,280 LB.

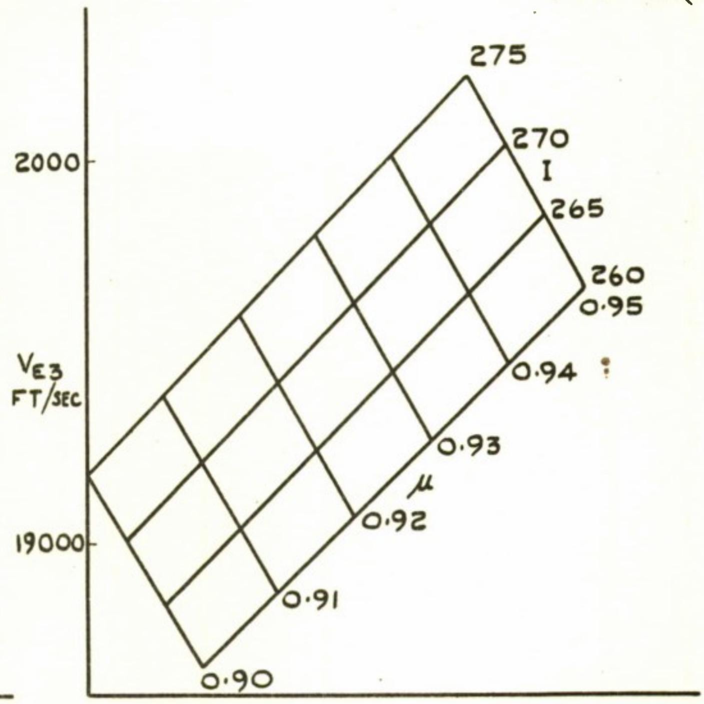


(d) PAYLOAD WEIGHT 110 LB. WT.
FIRST STAGE T = 25000 LB. 4½' DIA. TANKS
I = 220 W_p = 15,600 LB. W_E = 1480 LB.

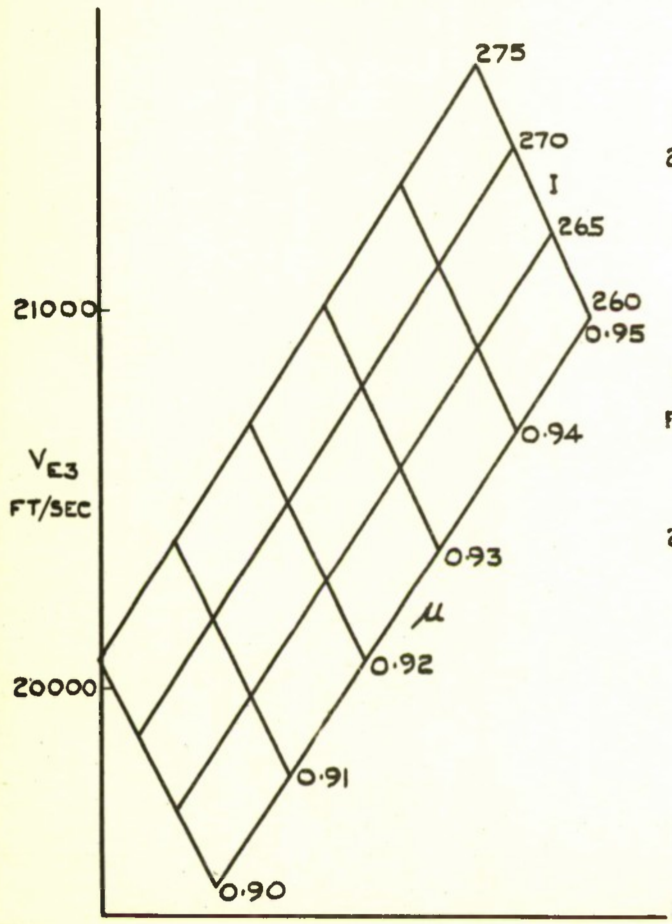
FIG. 14. (a – d) RE-ENTRY VELOCITY V_{E_3} AS A FUNCTION OF MASS FRACTION μ & PROPELLANT WEIGHT W_p OF THE SECOND STAGE ROCKET FOR A RANGE OF THREE STAGE CONFIGURATIONS.



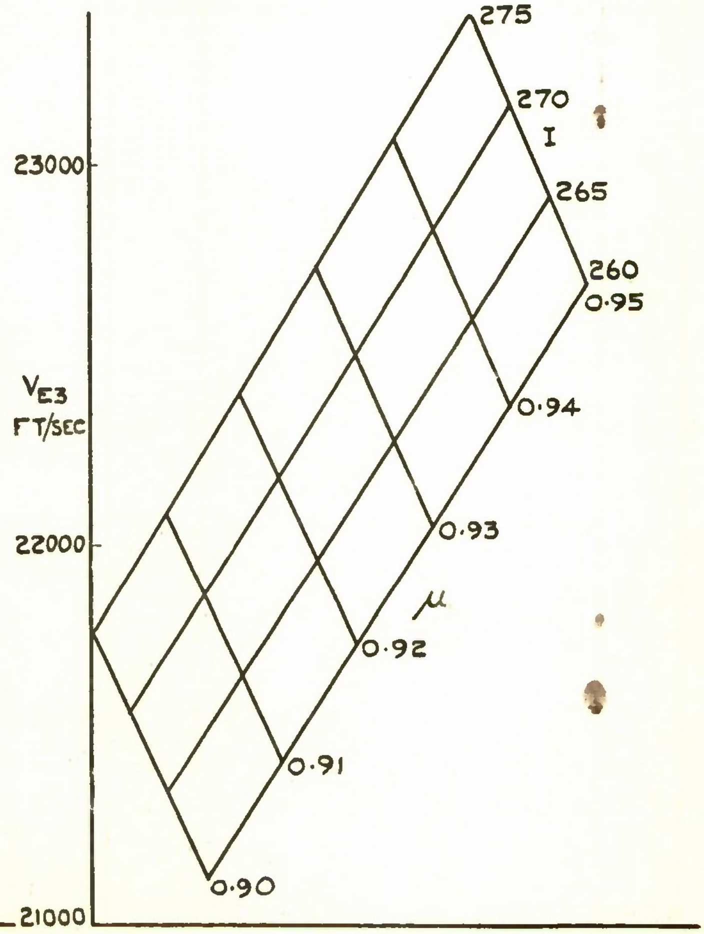
(a) PAYLOAD WEIGHT 250 LB.
FIRST STAGE T = 21,600 LB. WT.
I = 213, W_P = 11600 LB. W_E = 1380 LB.



(b) PAYLOAD WEIGHT 250 LB.
FIRST STAGE T = 25,000 LB. WT. 4½' DIA. TANKS
I = 220, W_P = 15600 LB. W_E = 1480 LB.



(c) PAYLOAD WEIGHT 110 LB.
FIRST STAGE T = 21,600 LB. WT.
I = 213, W_P = 11600 LB. W_E = 1380 LB.



(d) PAYLOAD WEIGHT 110 LB.
FIRST STAGE T = 25,000 LB. WT. 4½' DIA. TANKS
I = 220, W_P = 15600 LB. W_E = 1480 LB.

FIG. 15.(a-d) RE-ENTRY VELOCITY V_{E3} AS A FUNCTION OF MASS FRACTION μ AND SPECIFIC IMPULSE I OF THE SECOND STAGE ROCKET FOR A RANGE OF THREE STAGE CONFIGURATIONS.

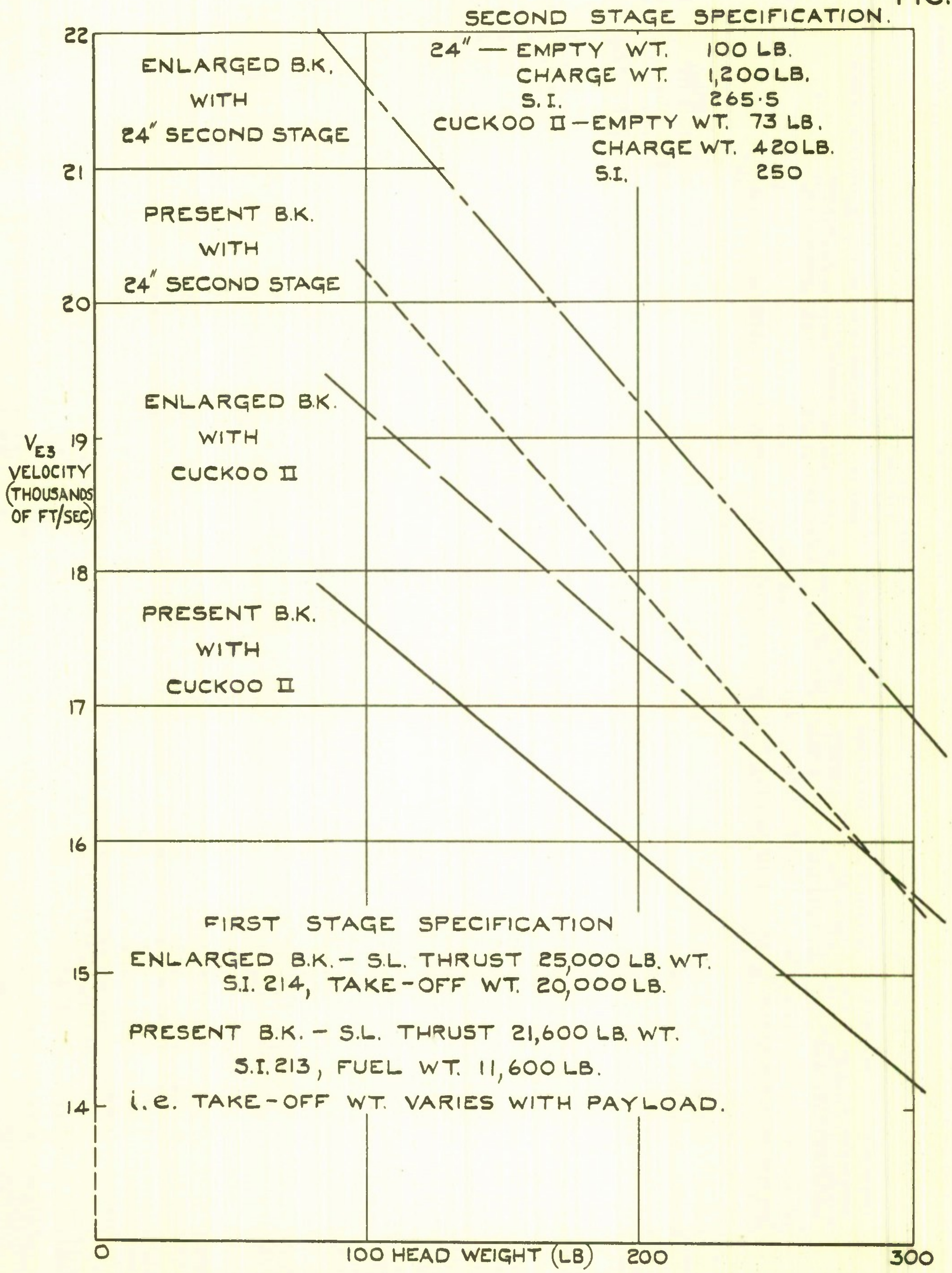
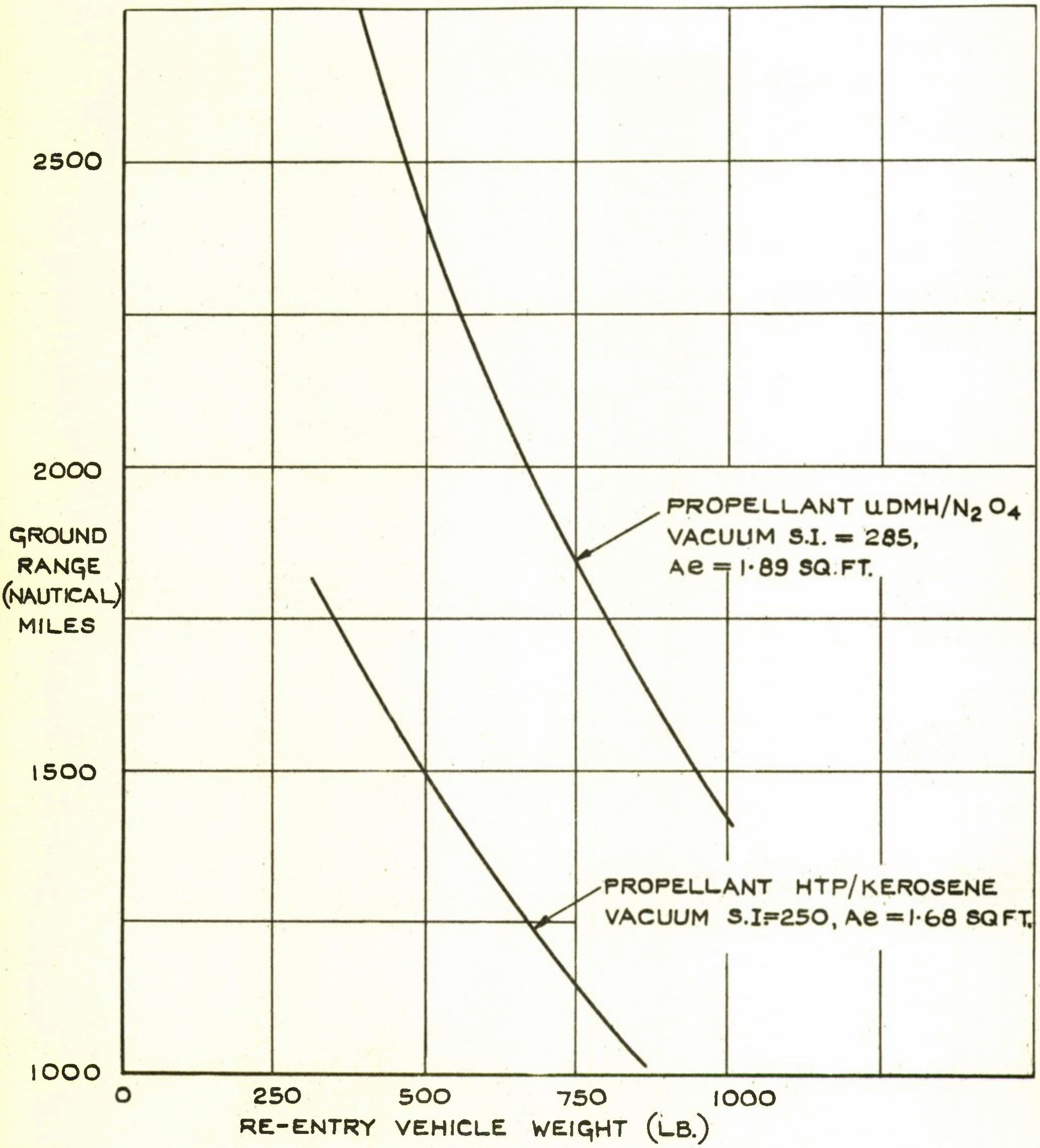


FIG.16. VARIATION OF HEAD RE-ENTRY VELOCITY V_{E3} OF BLACK KNIGHT WITH HEAD WEIGHT FOR DIFFERENT COMBINATIONS OF FIRST AND SECOND STAGE.

(FROM 4 TO 8 IMP. X., DEPENDING ON HEAD WEIGHT, ARE USED TO SEPARATE HEAD AND SECOND STAGE BY 20,000 FT AT 300,000 FT. ALTITUDE AT A SEPARATION VELOCITY OF UP TO 700 FT/SEC.

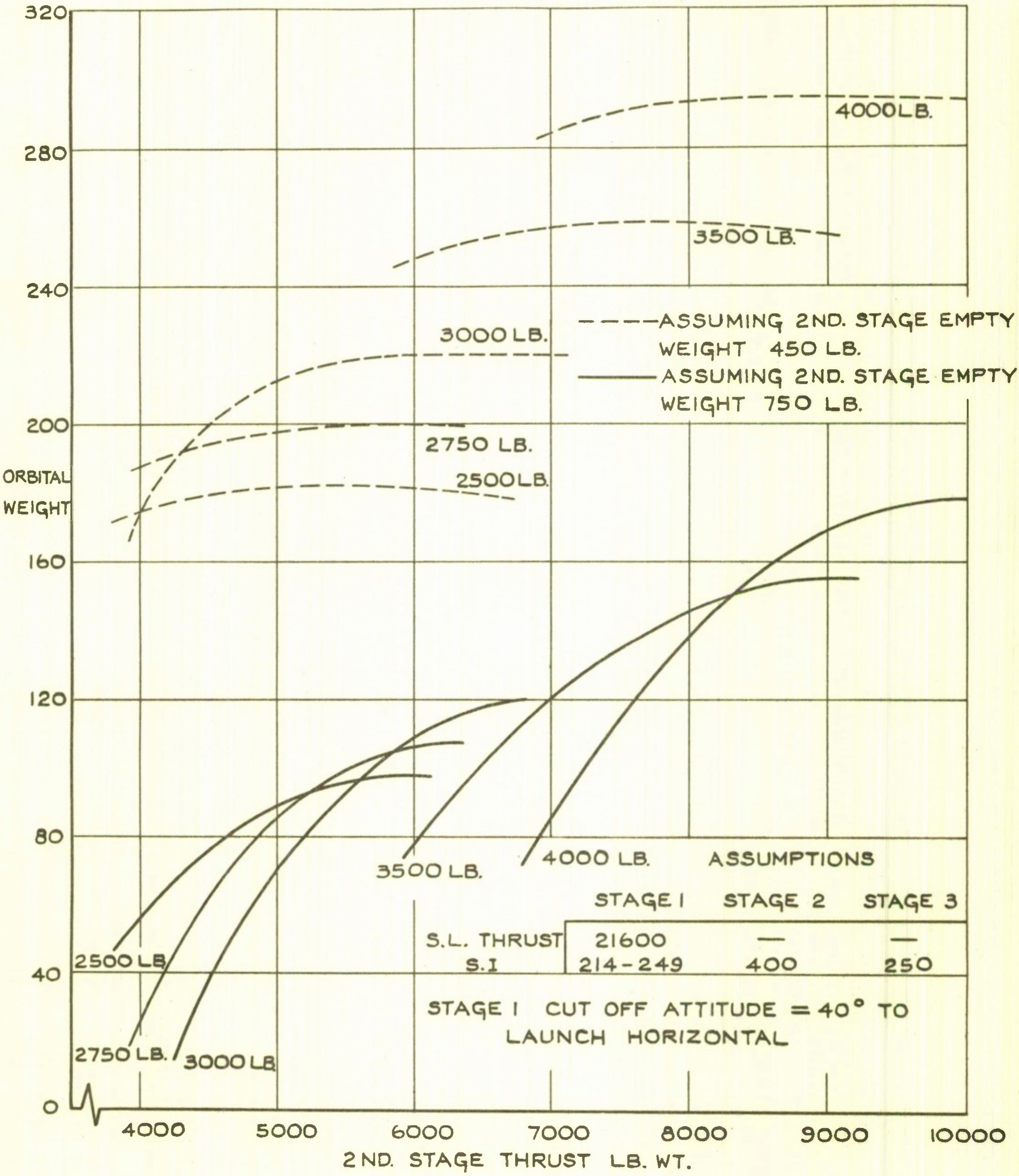
FIG. 17.



FIRST STAGE THRUST = 25 000 LB. WT. AT SEA LEVEL. TAKE OFF WEIGHT = 20000 LB.
SECOND STAGE THRUST = 6000 LB WT IN VACUO. WEIGHT = 3500 LB. & RE-ENTRY VEHICLE

FIG. 17. RANGE AS A FUNCTION OF REENTRY VEHICLE WEIGHT FOR A TWO STAGE LIQUID PROPELLANT VEHICLE WITH 4½ FT. DIA. TANKS.

FIG.18.



	STAGE I	STAGE 2	STAGE 3
S.L. THRUST	21600	—	—
S.I	214-249	400	250

FIG. 18. EFFECT OF SECOND STAGE THRUST ON ORBIT WEIGHT AT 300 N.M. FOR A THREE STAGE BLACK KNIGHT WITH LIQUID HYDROGEN SECOND STAGE.

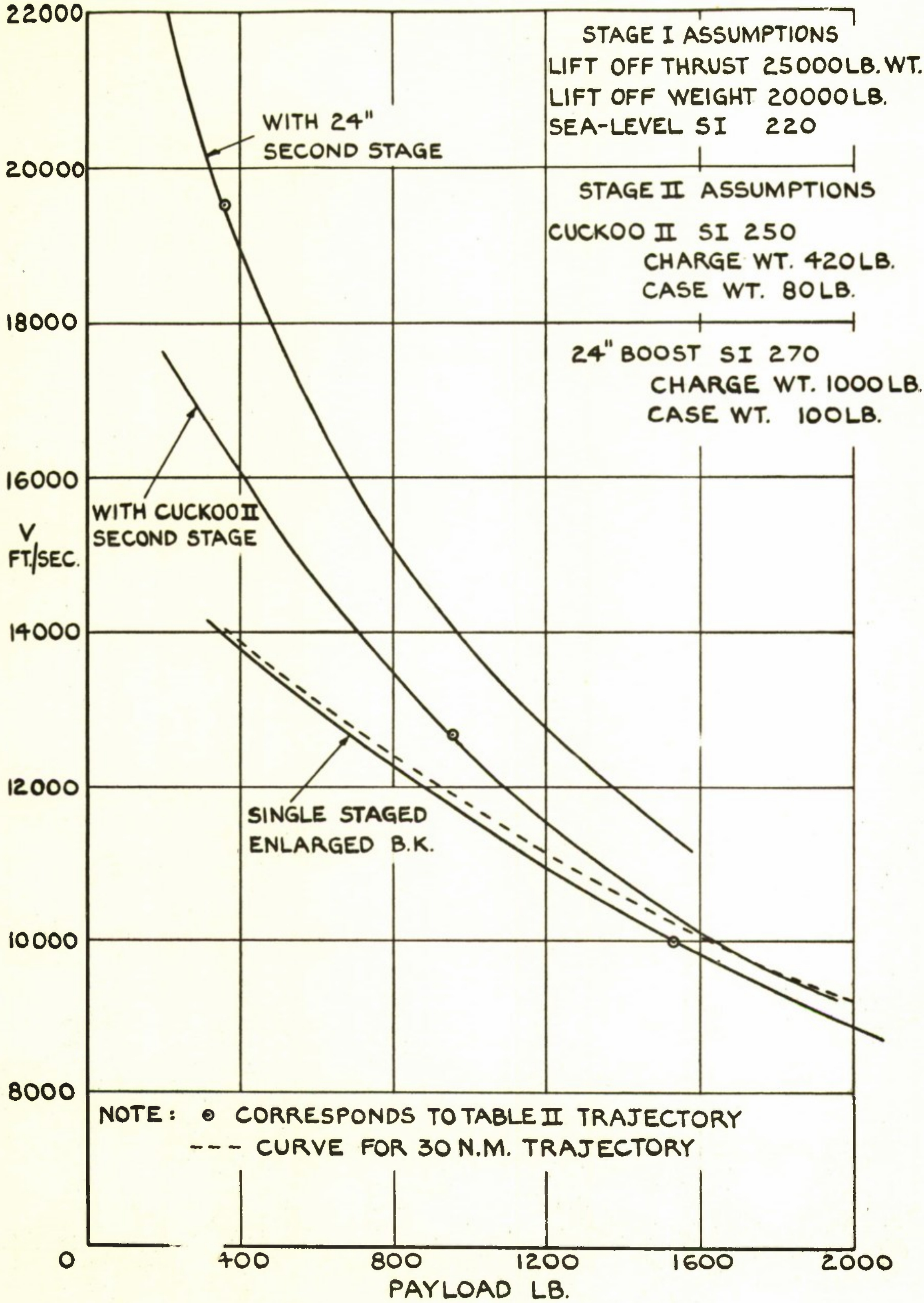
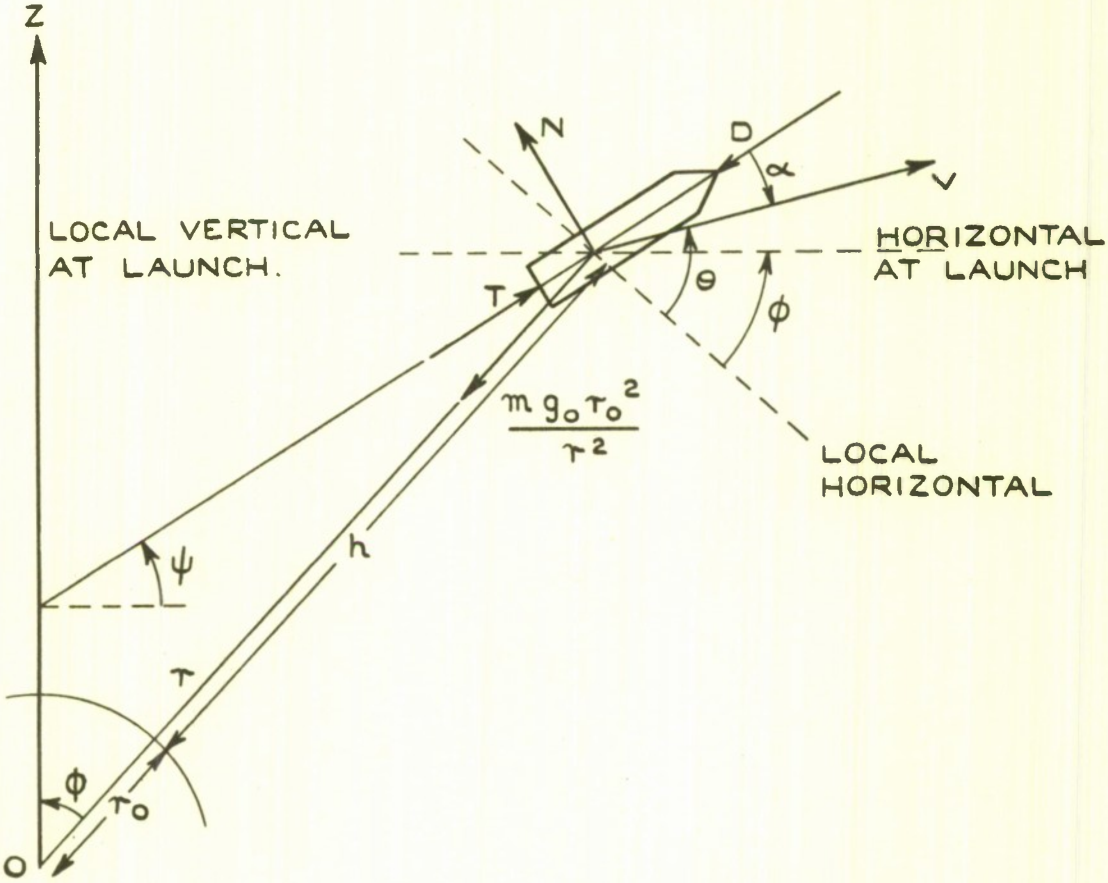
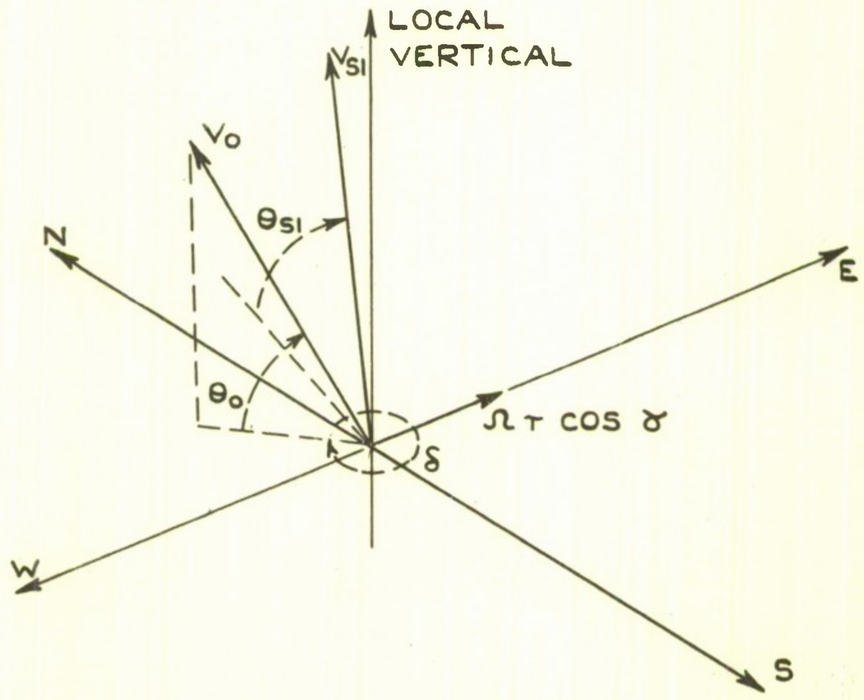


FIG. 19. HORIZONTAL VELOCITY V AT 50 N.M. ALTITUDE AS A FUNCTION OF PAYLOAD FOR 54 INCH BLACK KNIGHT WITH AND WITHOUT A SECOND STAGE.

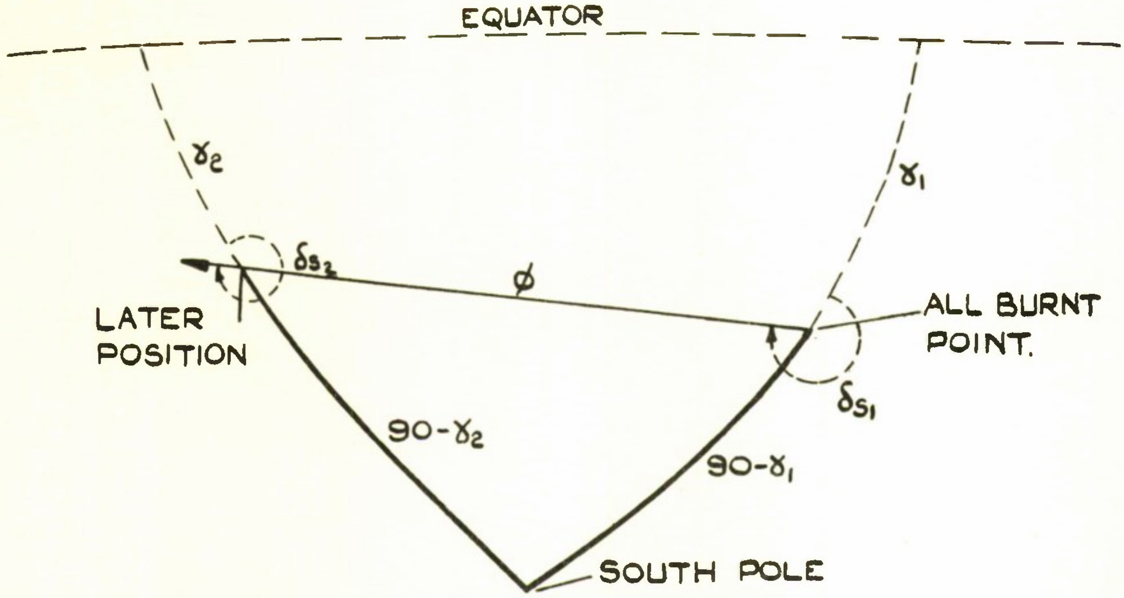


(a) DEFINITION OF NOTATION AND FORCES ON MISSILE.

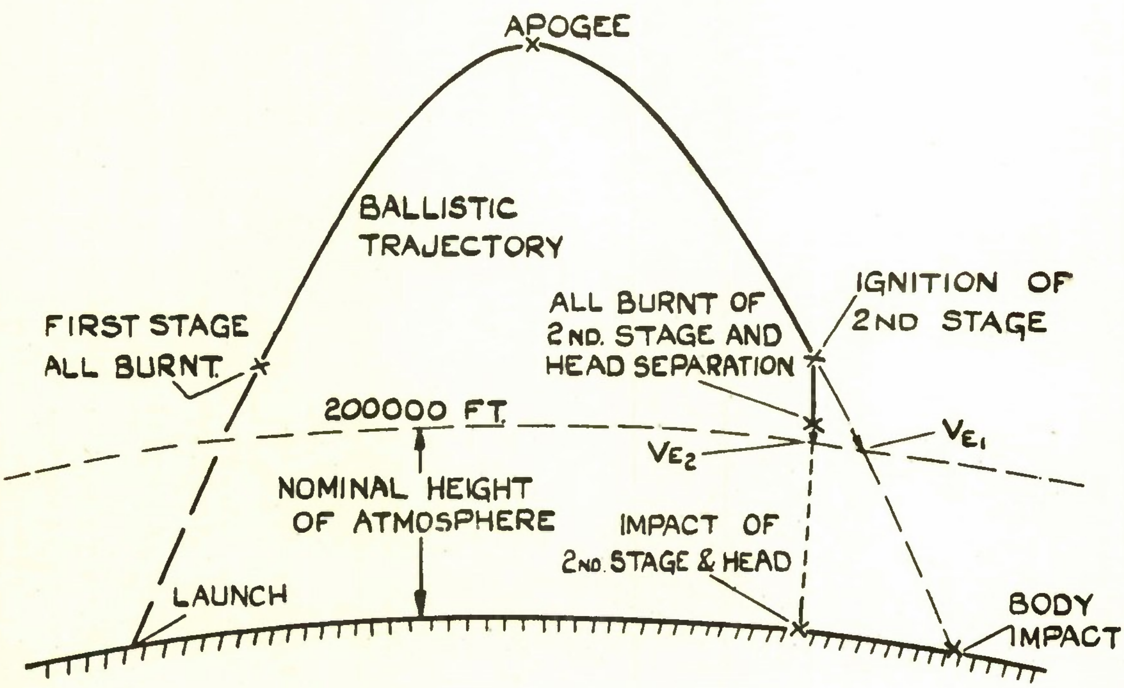


(b) RELATIONSHIP OF EARTH FIXED AND SPACE AXES.

FIG. 20. (a & b) DEFINITION OF CO-ORDINATES.



(a) PLAN OF FLIGHT IN SPACE



(b) ELEVATION OF FLIGHT IN SPACE OF TWO STAGE VEHICLE.

FIG. 21. (a & b) PLAN AND ELEVATION VIEWS OF FLIGHT PATH.

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Technical Note No. Space 60 — 623.451-519.074(41) Black Knight:
Royal Aircraft Establishment — 629.192.21.074(41) Black Knight

THE PERFORMANCE CAPABILITY OF THE BLACK KNIGHT ROCKET IN VARIOUS ROLES.
Waterfall, A.P. and Parkin, L.W. March 1964.

Performance studies made over a number of years for Black Knight are collected together in one document. Most of these calculations are concerned with the vehicle used as a re-entry test vehicle or as a sounding rocket. The note summarises the results and serves to illustrate the improvements in performance which have been made; results are presented in a way which enables the effects on performance of changes in the basic vehicle characteristics to be readily predicted.

Also included are the results of specific studies made to determine the feasibility of using Black Knight as a ballistic missile, as the first stage of a satellite launcher or as a launcher for a hypersonic glider.

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[dstl]

Information Centre
Knowledge Services
[dstl] *Porton Down,*
Salisbury
Wilts
SP4 0JQ
22060-6218
Tel: 01980-613753
Fax 01980-613970

Defense Technical Information Center (DTIC)
8725 John J. Kingman Road, Suit 0944
Fort Belvoir, VA 22060-6218
U.S.A.

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